Topographic form stress in the Southern Ocean State Estimate

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Context

• Theories of the ACC suggest that without boundaries in Drake Passage latitudes, zonal momentum input by wind stress to the ACC is balanced by topographic form stress (The papers we have discussed in this seminar: Munk and Palmen, Johnson and Bryden, Hughes, Olbers)

• Idealized models and high resolution GCM’s have demonstrated that wind stress balances form stress in the zonal integral (McWilliams et al. (1978), Treguier and McWilliams (1990), Wolff et al. (1991), and Marshall et al. (1993), Killworth and Nanneh (1994) and Stevens and Ivchenko (1997))

• Gille et al. (1997), looked at the spatial distribution of form stress in 10 degree longitude sections and found that form stress balanced wind stress at all latitudes
Total vertically integrated form stress integrated along streamlines in 10 degree wide swaths

Gille et al. 1997 Fig. 6
This work

- Analyse output from an eddy-permitting data-assimilating state estimate of the Southern Ocean

- Use a direct approach to calculate the pressure gradient across every piece of topography in the domain to obtain full 2-D map of topographic form stress
The Southern Ocean State Estimate

• MITgcm least squares fit to observations using adjoint method
• eddy-permitting, 1/6° resolution, 42 vertical levels, 900 s time step
• data include Argo, CTD, MEOP, satellite SSH, SST, ice cover and geoid
• ERA-interim buoyancy and wind forcing, atmospheric state is adjusted to be consistent with assimilated ocean observations
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Zonal momentum equation (steady state, continuity):

$$
\rho_0 \frac{\partial}{\partial x} \left( \bar{u} \bar{u} \right) + \rho_0 \frac{\partial}{\partial y} \left( \bar{u} \bar{v} \right) + \rho_0 \frac{\partial}{\partial z} \left( \bar{u} \bar{w} \right) - \rho_0 f \bar{v} = - \frac{\partial \bar{p}}{\partial x} + \frac{\partial \bar{\tau}^x}{\partial z} + \rho_0 \eta \nabla^2 \bar{u} \quad (A1)
$$

Circumpolar and vertical integral:

$$
\rho_0 \int_x \frac{\partial}{\partial y} \int_{z=-H}^{\eta} \bar{u} \bar{v} \, dz \, dx = - \int_x \int_{z=-H}^{\eta} \frac{\partial \bar{p}}{\partial x} \, dz \, dx + \int_x \bar{\tau}_{\text{wind}}^x \, dx - \int_x \bar{\tau}_{\text{friction}}^x \, dx. \quad (1)
$$
Vertically integrated momentum balance terms

wind stress

vertically integrated zonal pressure gradient

friction

meridional momentum flux divergence

\[
\rho_0 \left( \int_x \frac{\partial}{\partial y} \int_{z=-H}^{\eta} \bar{u}\bar{v} \, dz \, dx \right) - \int_x \int_{z=-H}^{\eta} \frac{\partial \bar{p}}{\partial x} \, dz \, dx + \int_x \tau_{wind}^x \, dx - \int_x \tau_{friction}^x \, dx.
\]

(1)
Can divide pressure gradient term:

\[- \int_x \int_{z=-H}^{\eta} \frac{\partial p}{\partial x} d\bar{z} \, dx = - \int_x \frac{\partial}{\partial x} \int_{z=-H}^{\eta} \bar{p} \, d\bar{z} \, dx + \int_x p_{\text{atm}} \frac{\partial \eta}{\partial x} \, dx + \int_x p_b \frac{\partial H}{\partial x} \, dx, \tag{2}\]

- net pressure gradient across zonally bounded basin
- transfer of zonal momentum from atmosphere to fluid (negligibly small)
- transfer of zonal momentum from fluid to solid earth (topographic form stress?)
Can divide pressure gradient term:

\[
- \int_{x} \int_{z=-H}^{\eta} \frac{\partial \bar{p}}{\partial x} dz dx = - \int_{x} \frac{\partial}{\partial x} \int_{z=-H}^{\eta} \bar{p} dz dx + \int_{x} \rho_{atm} \frac{\partial \eta}{\partial x} dx + \int_{x} \rho_{b} \frac{\partial H}{\partial x} dx.
\]  

"Total form stress"

- net pressure gradient across zonally bounded basin
- transfer of zonal momentum from atmosphere to fluid (negligibly small)
- transfer of zonal momentum from fluid to solid earth (topographic form stress?)
Discretize zonal pressure gradient term to represent distribution of total form stress:

\[-\int_{x}^{\tilde{\eta}} \int_{z=-H}^{\tilde{\eta}} \frac{\partial \bar{p}}{\partial x} dz dx = - \sum_{x} \sum_{z} \frac{\Delta \bar{p}}{\Delta x} \Delta z \Delta x = - \sum_{x} \sum_{z} \Delta \bar{p} \Delta z.\]
Discretize zonal pressure gradient term to represent distribution of total form stress:

\[- \int_{x}^{x_{	ext{end}}} \int_{z=-H}^{\eta} \frac{\partial \bar{p}}{\partial x} \, dz \, dx = - \sum_{x} \sum_{z} \frac{\Delta \bar{p}}{\Delta x} \Delta z \Delta x = - \sum_{x} \sum_{z} \Delta \bar{p} \Delta z.\]

In other words: contribution of each topographic feature to the zonally integrated pressure gradient
\[ \Delta \bar{\rho}_b = \bar{\rho}_b(x=x_E) - \bar{\rho}_b(x=x_W) \]
\[ \Delta \bar{p}_b = \bar{p}_b(x=x_E) - \bar{p}_b(x=x_W) \]

\[ \frac{\Delta \bar{p}_b}{\Delta x} = \frac{\bar{p}_b(x=x_E) - \bar{p}_b(x=x_W)}{x_E - x_W} \]
\[ \Delta \overline{p_b} = \overline{p_b}(x = x_E) - \overline{p_b}(x = x_W) \]

\[ \frac{\Delta \overline{p_b}}{\Delta x} = \frac{\overline{p_b}(x = x_E) - \overline{p_b}(x = x_W)}{x_E - x_W} \]

\[ \sum_{z=-H}^{\eta} \frac{\Delta \overline{p_b}}{\Delta x} \Delta z. \]
\[ \Delta \bar{\rho}_b = \bar{\rho}_b(x=x_E) - \bar{\rho}_b(x=x_W) \]

\[ \frac{\Delta \bar{p}_b}{\Delta x} = \frac{\bar{\rho}_b(x=x_E) - \bar{\rho}_b(x=x_W)}{x_E - x_W} \]

\[ \sum_{z=-H}^{\eta} \frac{\Delta \bar{p}_b}{\Delta x} \Delta z. \]

\[ \frac{\Delta p}{\Delta z} = \int_{x}^{x} \tau \, dx \]

\[ v = \frac{1}{\rho_0} \frac{\Delta p}{\Delta x} \]
zonally integrate to find total form stress signal

\[-\int \int \frac{\partial p}{\partial x} dz dx = \sum \sum \frac{\Delta p_b}{\Delta x} \Delta z \Delta x = \sum \sum \Delta p_b \Delta z.\]
Zonal and depth-integrated momentum budget, integrated over meridional domain

<table>
<thead>
<tr>
<th>y Domain</th>
<th>Wind Stress</th>
<th>Topographic Form Stress</th>
<th>Frictional Stress</th>
<th>Flux Divergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC latitudes 42°S to 65°S</td>
<td>6.67×10^{12} N (+100%)</td>
<td>−6.36×10^{12} N (−95%)</td>
<td>−0.19×10^{12} N (−3%)</td>
<td>−0.16×10^{12} N (−2%)</td>
</tr>
<tr>
<td>Full domain 30°S to 77°S</td>
<td>8.03×10^{12} N (+100%)</td>
<td>−7.30×10^{12} N (−91%)</td>
<td>−0.11×10^{12} N (−1%)</td>
<td>−0.61×10^{12} N (−8%)</td>
</tr>
</tbody>
</table>

^aPositive sign indicates eastward direction; negative sign indicates westward direction.
latitudinal distribution of zonal and depth-integrated terms

Fig. 3

\[ \int_{-x}^{x} \tau_{\text{wind}} \, dx \]

\[ \sum_{\text{ridges}} \sum_{z} \Delta p \Delta z \]

\[ \int_{-x}^{x} \tau_{\text{friction}} \, dx \]

\[ -\rho_0 \int_{-y}^{y} \frac{\partial}{\partial y} \int_{z} (uw) \, dz \, dx \]

Sum

Latitude

Zonal and depth integrated stress [N/m] \( \times 10^6 \)
spatial distribution of wind stress and TFS
spatial distribution of TFS

full depth

0-3700 m

3700 m - bottom

Fig. 4
depth distribution of TFS for different latitude ranges

Fig. 5
zonal cumulative sum

Fig. 6
Fig. 8

Abyssal TFS
Temporal variability

Fig. 9

(a) Temporal variability of xy-integrated stress over ACC latitudes [N].

\[-1 \times \text{wind stress } \int_y \phi_x \tau_{wind}^x \text{dxdy} \quad \text{Form stress } \sum_y \sum_{ridges} \sum_{z=0} H \Delta \bar{p} \Delta z \Delta y\]

(b) Percent variance explained over lag [days].
Temporal variability

a) standard deviation of $\sum_{z}^{\eta} - H \frac{\Delta P_h}{\Delta x} \Delta z$

b) $\sigma_{SOSE\ adjustment}^2 / \sigma_{ERA-Interim}^2$
Conclusions

• 95% of the zonal momentum input via wind stress at the surface is balanced by topographic form stress across ocean ridges, while the remaining 5% is balanced via bottom friction and momentum flux divergences at the northern and southern boundaries.

• Nearly 40% of topographic form stress occurs across South America, while the remaining 60% occurs across the major submerged ridges that underlie the ACC.

• Shallow form stress in the top 3700 m balances most of the wind stress.

• 88% of the variance in the 6 year form stress time series can be explained by the wind stress signal, suggesting that changes in the integrated wind stress signal are communicated via rapid barotropic response down to the level of bottom topography.
Questions?