Why Western Boundary Currents in Realistic Oceans are Inviscid: A Link between Form Stress and Bottom Pressure Torques

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ABSTRACT

It is shown that wind stress curl is balanced by bottom pressure torque in a zonal integral over any strip wide enough to smooth out the effect of nonlinear terms (typically about 3° of latitude). The derivation is completely general as long as the zonal wind stress is balanced by form stress at each latitude, as is known to be the case in the ocean. This implies that viscous torques are not important in western boundary currents, their place being taken by bottom pressure torques. The prediction is confirmed in the context of a global, eddy-permitting, numerical ocean model. This link between form stress and bottom pressure torques makes it easier to consider Southern Ocean dynamics and subtropical gyre dynamics in the same conceptual framework, with topographic interactions being important in both cases.

Problem Setup

- Assume zonal wind stress
- Assume sloping side walls
- Assume Zonal wind stress is balanced by form stress

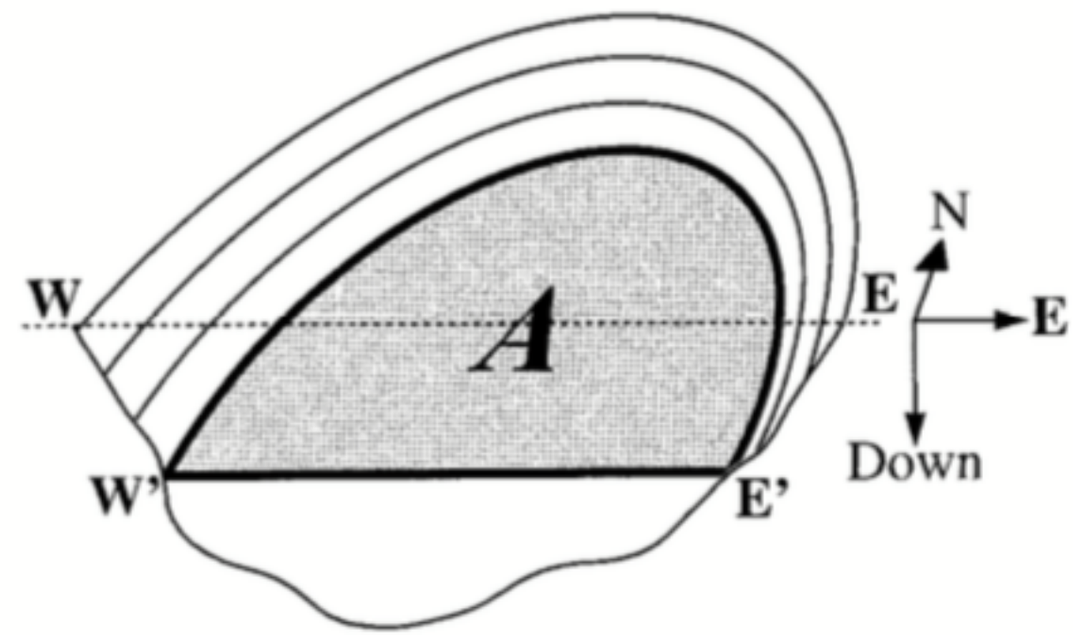

FIG. 1. Schematic to show a region of an ocean basin, cut off at a zonal section that includes surface points **W**, **E** and points **W′**, **E′** on the constant depth surface ***A***.

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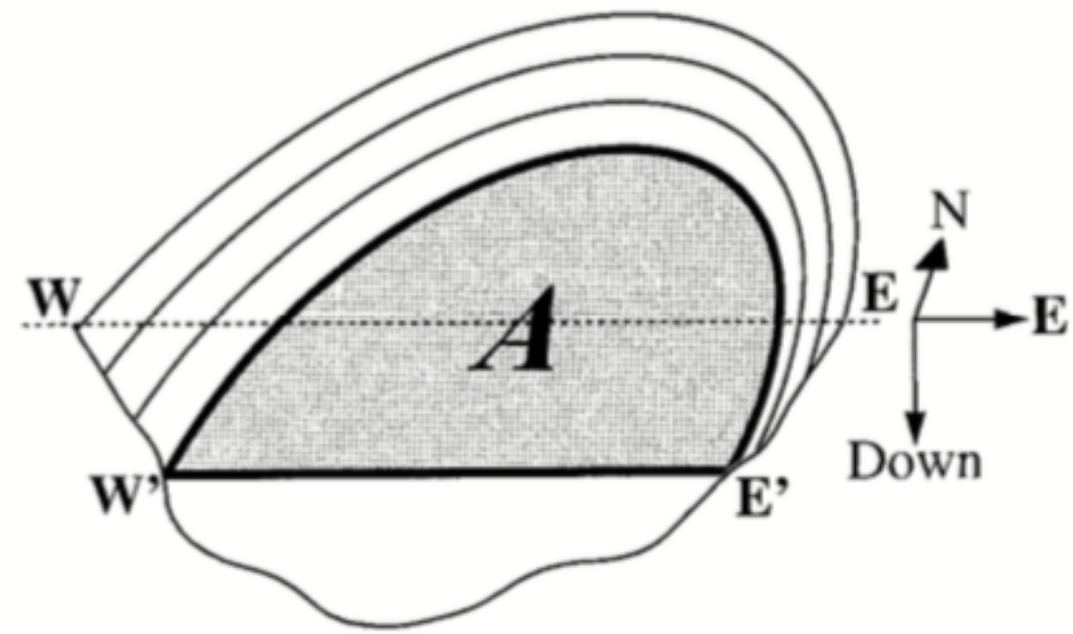

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Is this a fair assumption??

Review of Classical Boundary Layer Balances

$$\rho f \mathbf{k} \times \mathbf{u} = -\nabla p + \boldsymbol{\tau}_z + \mathbf{a} + \mathbf{b}, \qquad (1)$$

where f is the Coriolis parameter, $\mathbf{u}$ is the two-dimensional horizontal velocity, ρ is density, p is pressure, $\boldsymbol{\tau}$ is the viscous stress on a horizontal surface, $\mathbf{a}$ is the divergence of the remaining (lateral) viscous stress, and $\mathbf{b}$ represents terms nonlinear in $\mathbf{u}$.

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In a realistic ocean, with sloping sidewalls, the depth of the ocean goes to zero at its lateral boundaries ($-H = \eta$). At such boundaries, $p_b = p_a = 0$ and, being depth integrals of finite quantities, $\mathbf{U}$, P, $(-\nabla P + p_b \nabla H)$, $\mathbf{A}$, and $\mathbf{B}$ all go to zero.

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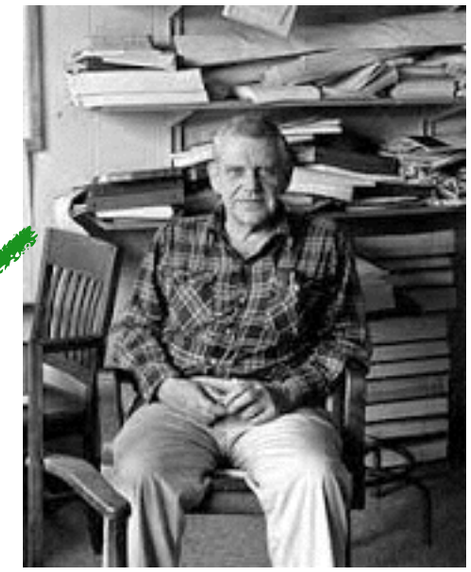

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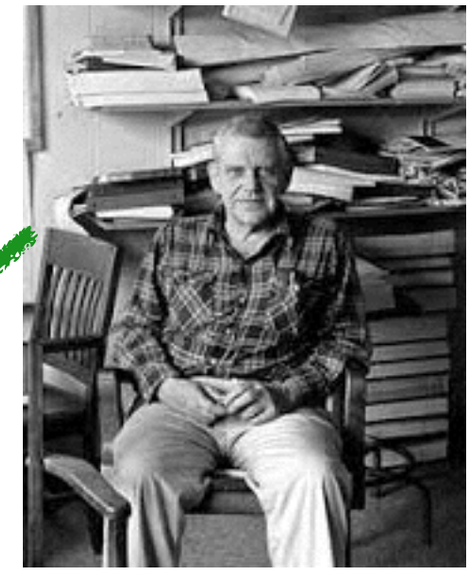

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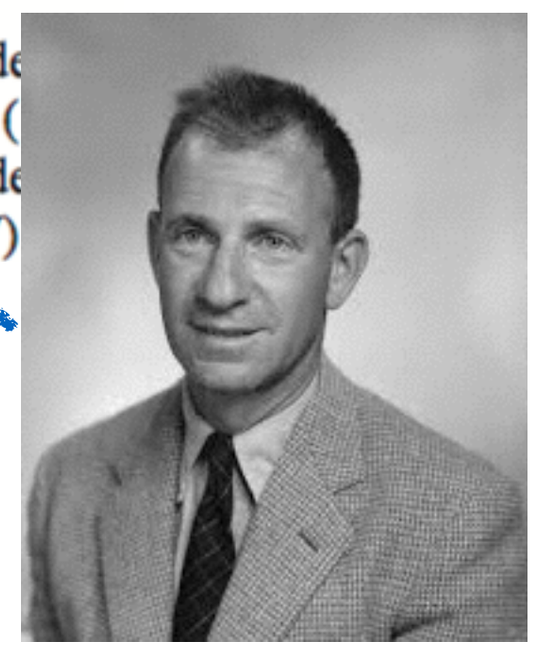

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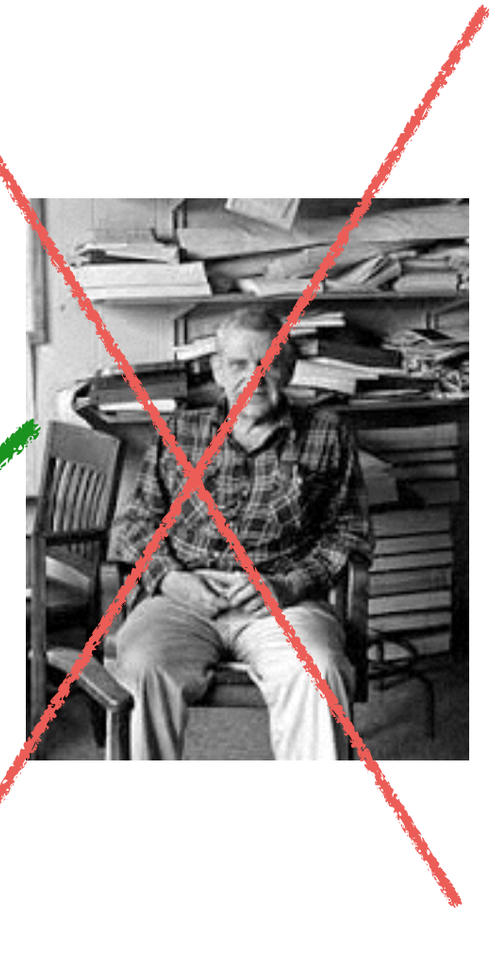

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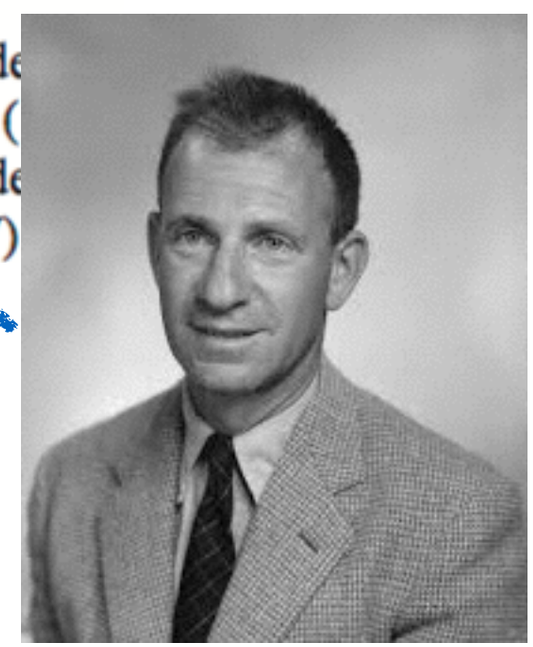

$$\int_A \nabla \times (p_b \nabla H)\, dS = \oint_{\delta A} p_b \nabla H \cdot \mathbf{ds}. \qquad (8)$$

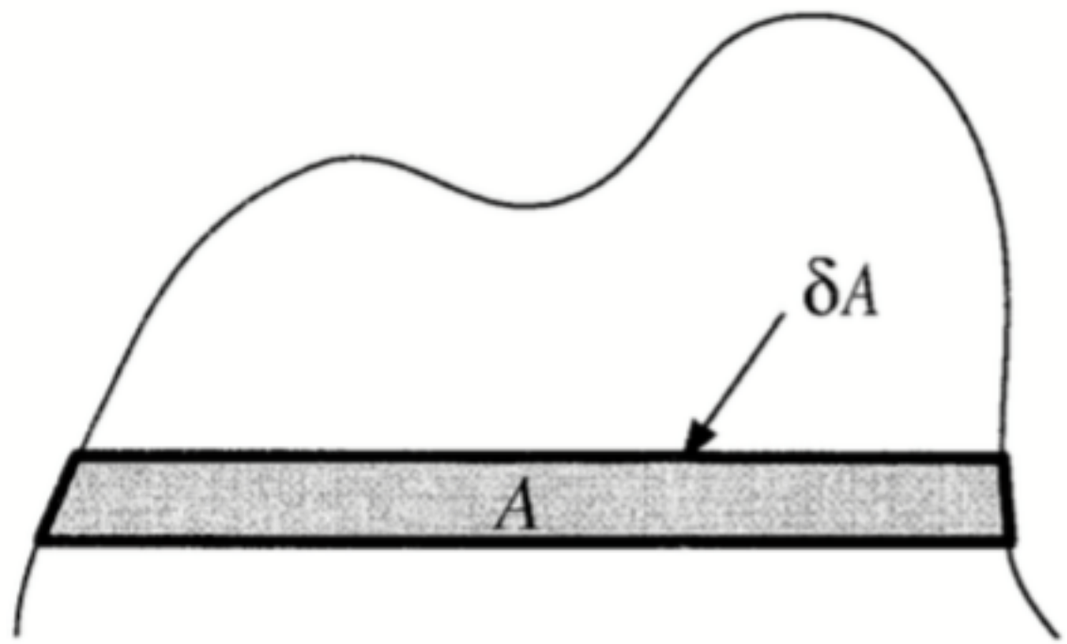

FIG. 2. Schematic to show an area $\boldsymbol{A}$, and its bounding curve $\delta\boldsymbol{A}$, which consists of two latitude lines and two stretches of coastline.

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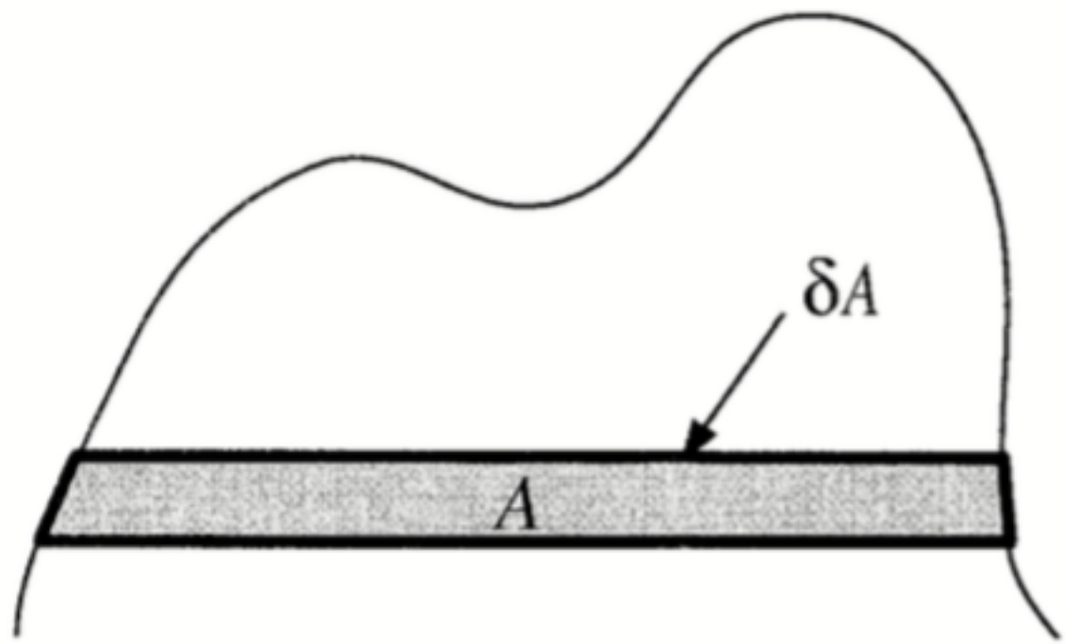
$$\int_A \nabla \times (p_b \nabla H)\, dS$$

$$= \int_{\phi_1} p_b H_x\, dx - \int_{\phi_2} p_b H_x\, dx, \qquad (9)$$

$$\int_A \nabla \times \boldsymbol{\tau}_0\, dS = \int_{\phi_1} \tau_0^x\, dx - \int_{\phi_2} \tau_0^x\, dx. \qquad (10)$$



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It therefore becomes clear from (9) and (10) that, if the zonal (wind − bottom) stress is balanced by form stress in zonal integrals at each latitude, then the area integral of (wind − bottom) stress curl is balanced by the area integral of bottom pressure torque for all areas bounded by latitude lines. If we consider an infinitesimal area bounded by two latitude lines separated by a distance δy, the area integral reduces to δy times the zonal line integral of terms in the BV equation (6), showing that we can write the line integral of each of these terms as d/dy of the corresponding term in the angular momentum balance (5). Since we know that the primary balance in (5) is between wind stress and form stress, we must conclude that wind stress curl is balanced by bottom pressure torque in the area integral of (6).

Or

???

The Model

- Ocean Circulation and Climate Advanced Modeling project model
- 0.25° model with 36 depth levels (255m-5500m)
- Relaxation towards Levitus 94 climatology for 4 years, then run forward for 4 years.

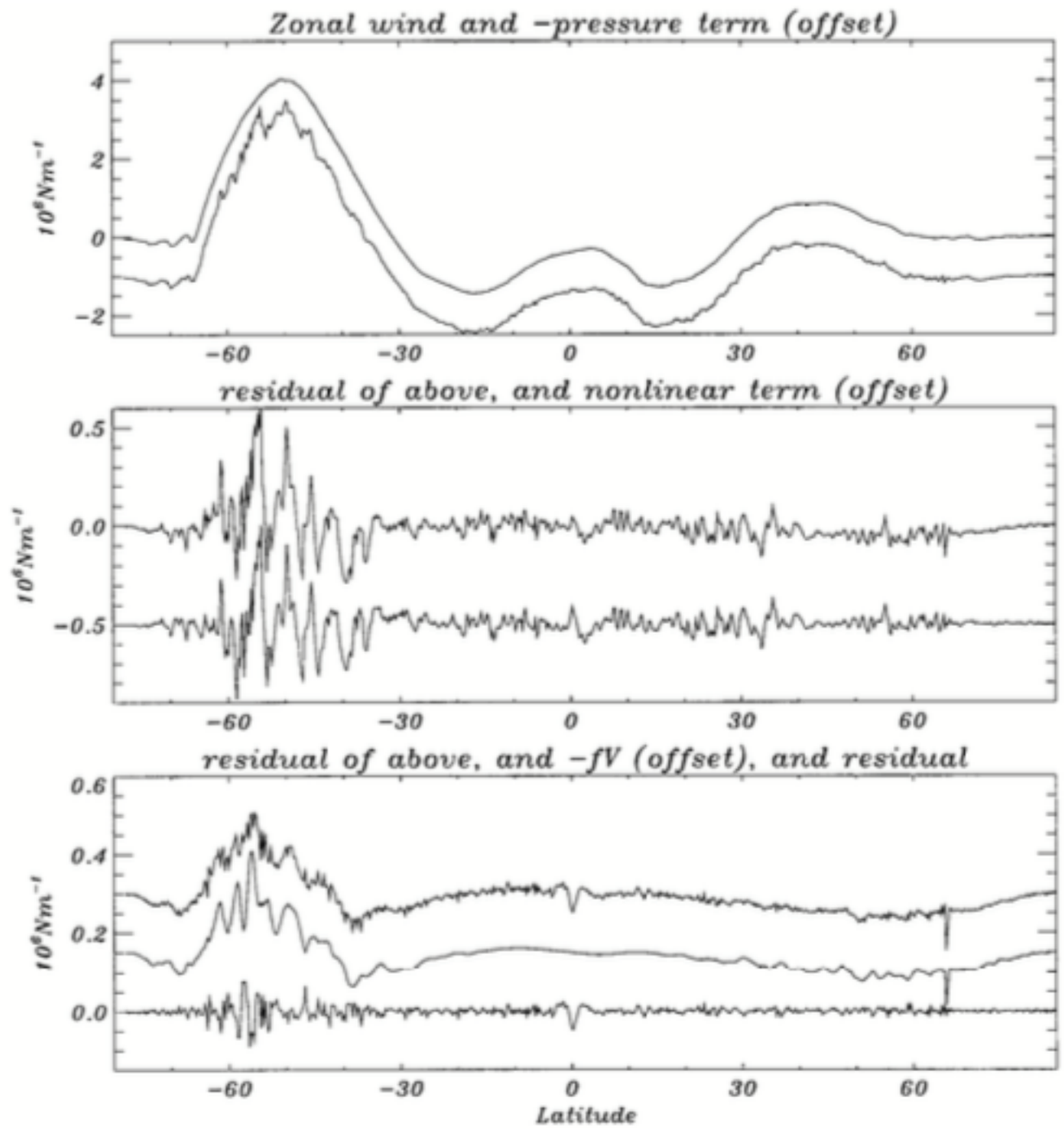
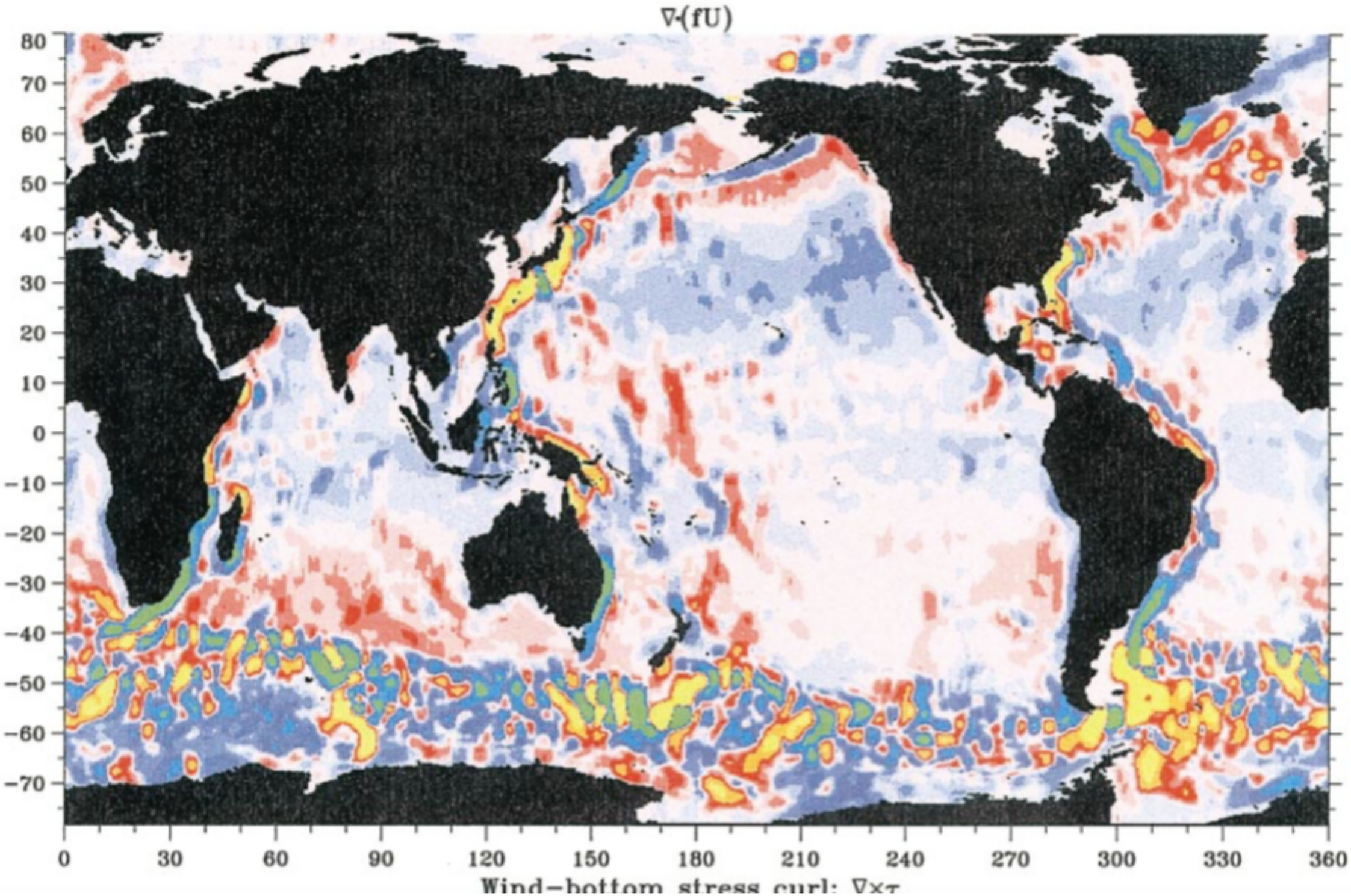


FIG. 3. Terms in the angular momentum balance (5) from OCCAM. The curves plotted are zonal integrals of, from top to bottom, τ^x (wind − bottom stress), $-p_b H_x$ (−form stress), $-(\tau^x + p_b H_x)$, b^x (nonlinear), $-(\tau^x + p_b H_x + b^x)$, $-fV$ (Coriolis), $-(\tau^x + p_b H_x + b^x + fV)$. See text for full definitions. Divide by $f\rho$ to get equivalent volume transports.

$$\nabla \cdot (f\mathbf{U}) = \nabla \times (p_b \nabla H) + \nabla \times \tau_0 + \nabla \times (\mathbf{A} + \mathbf{B}). \qquad (6)$$



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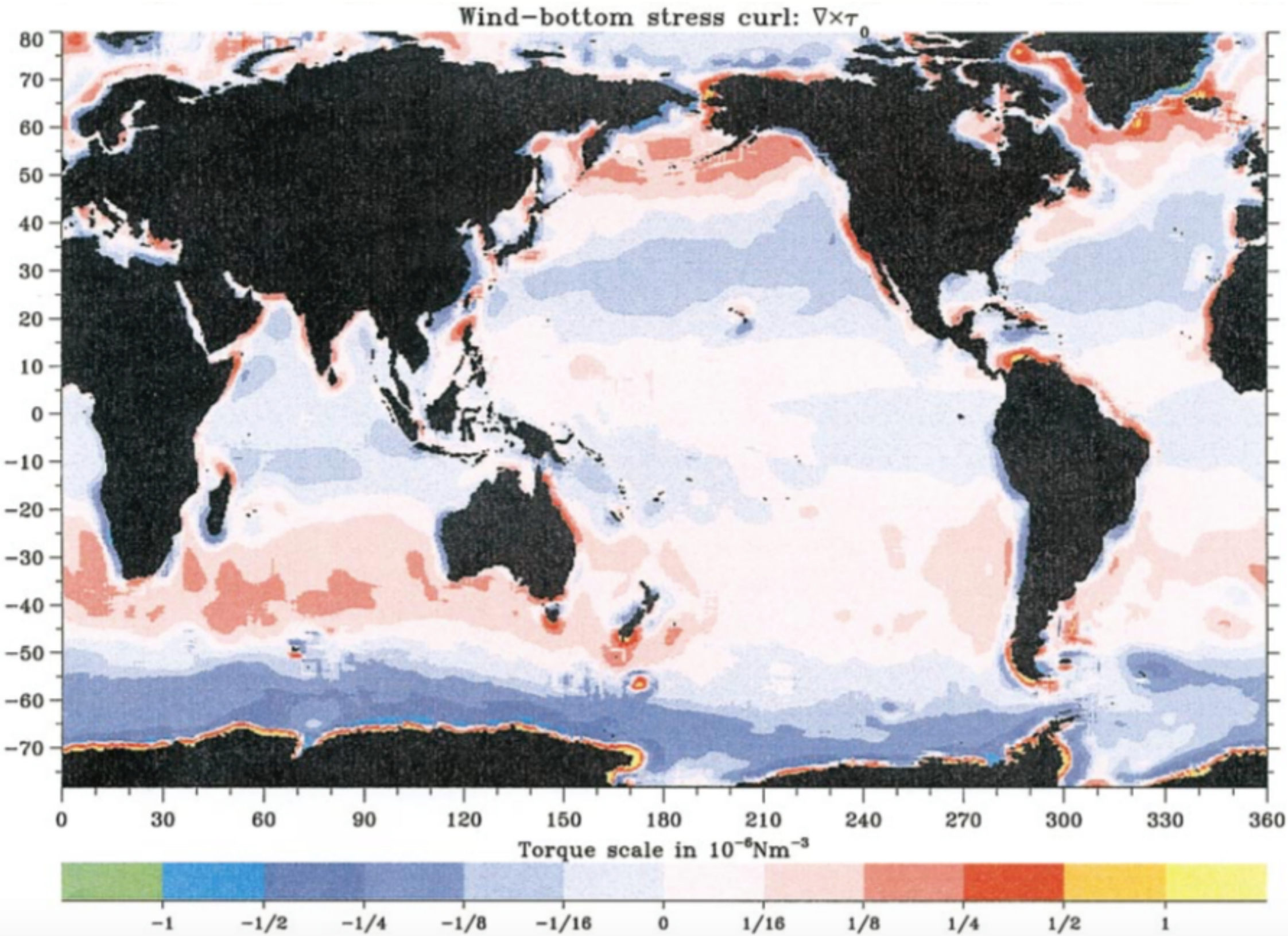

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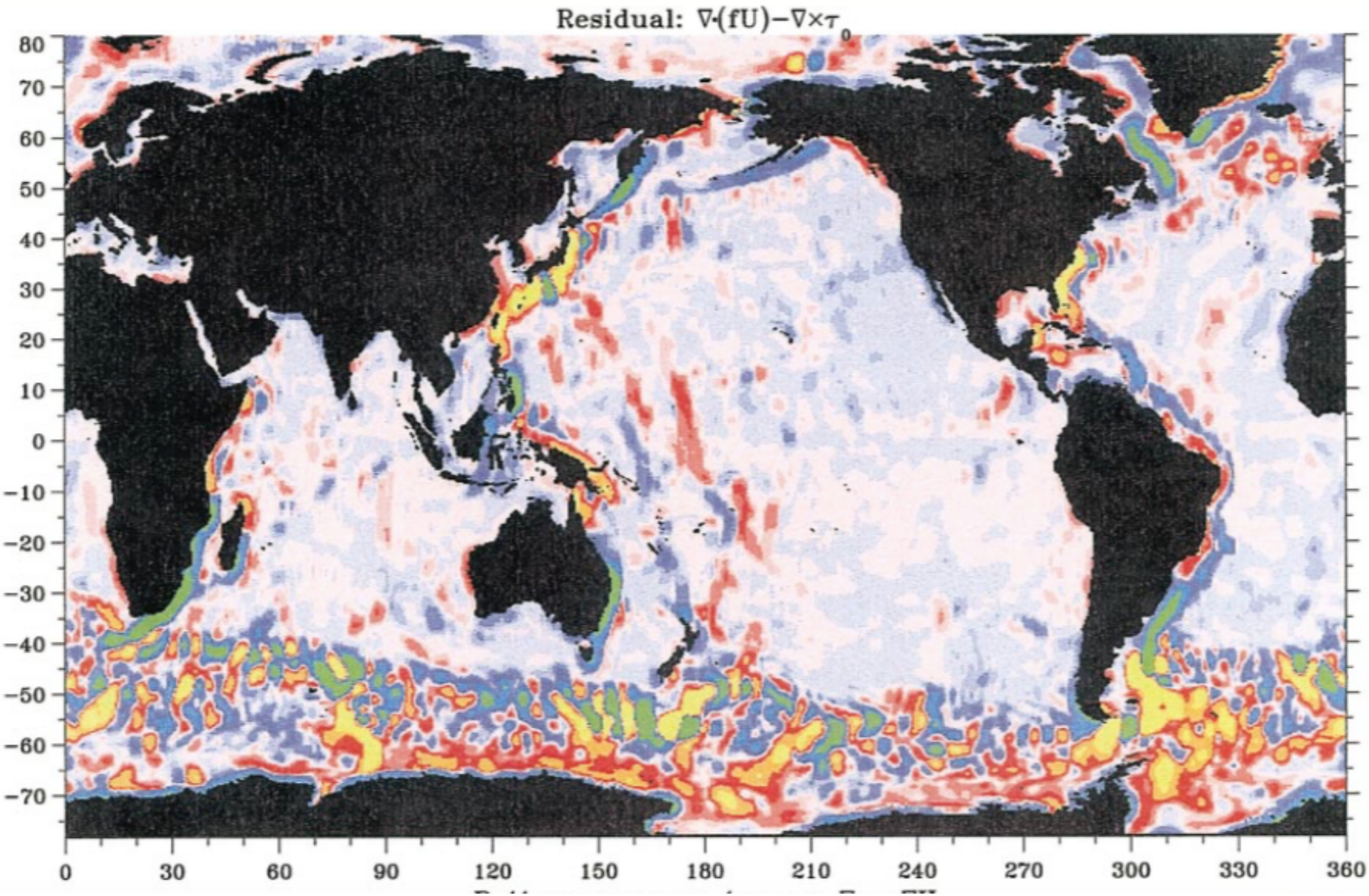

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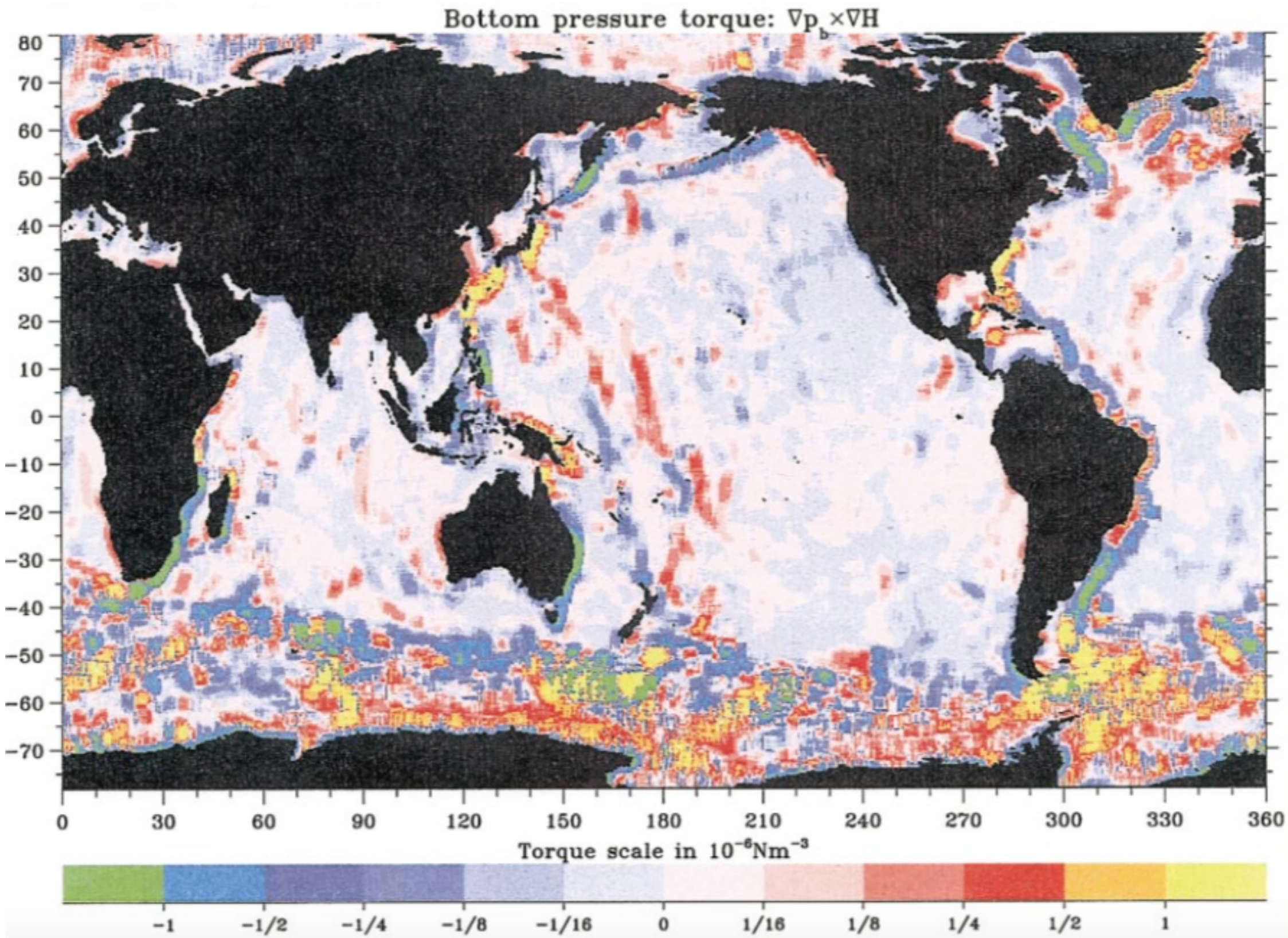

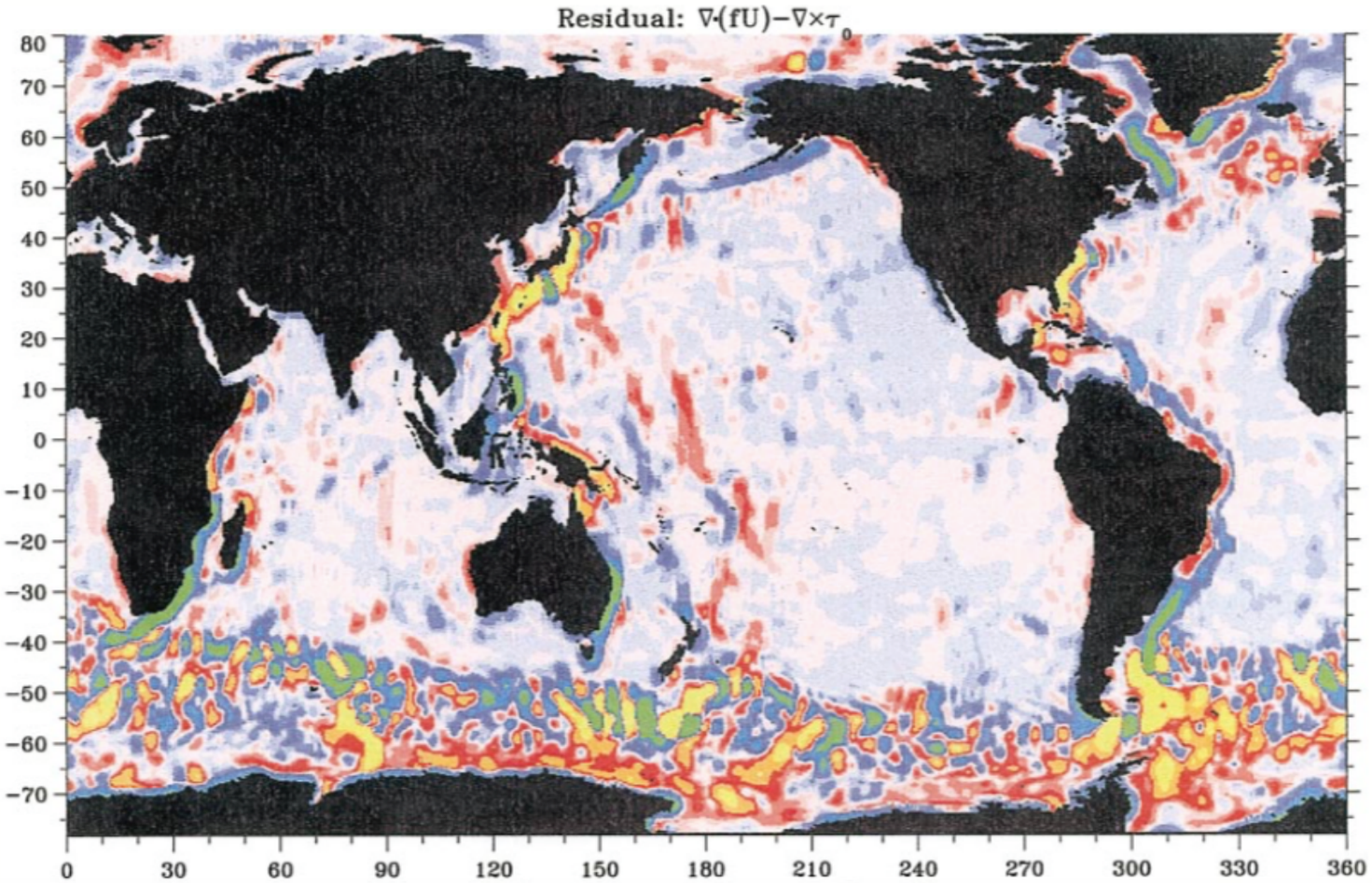
Residual: ∇·(fU)−∇×τ0
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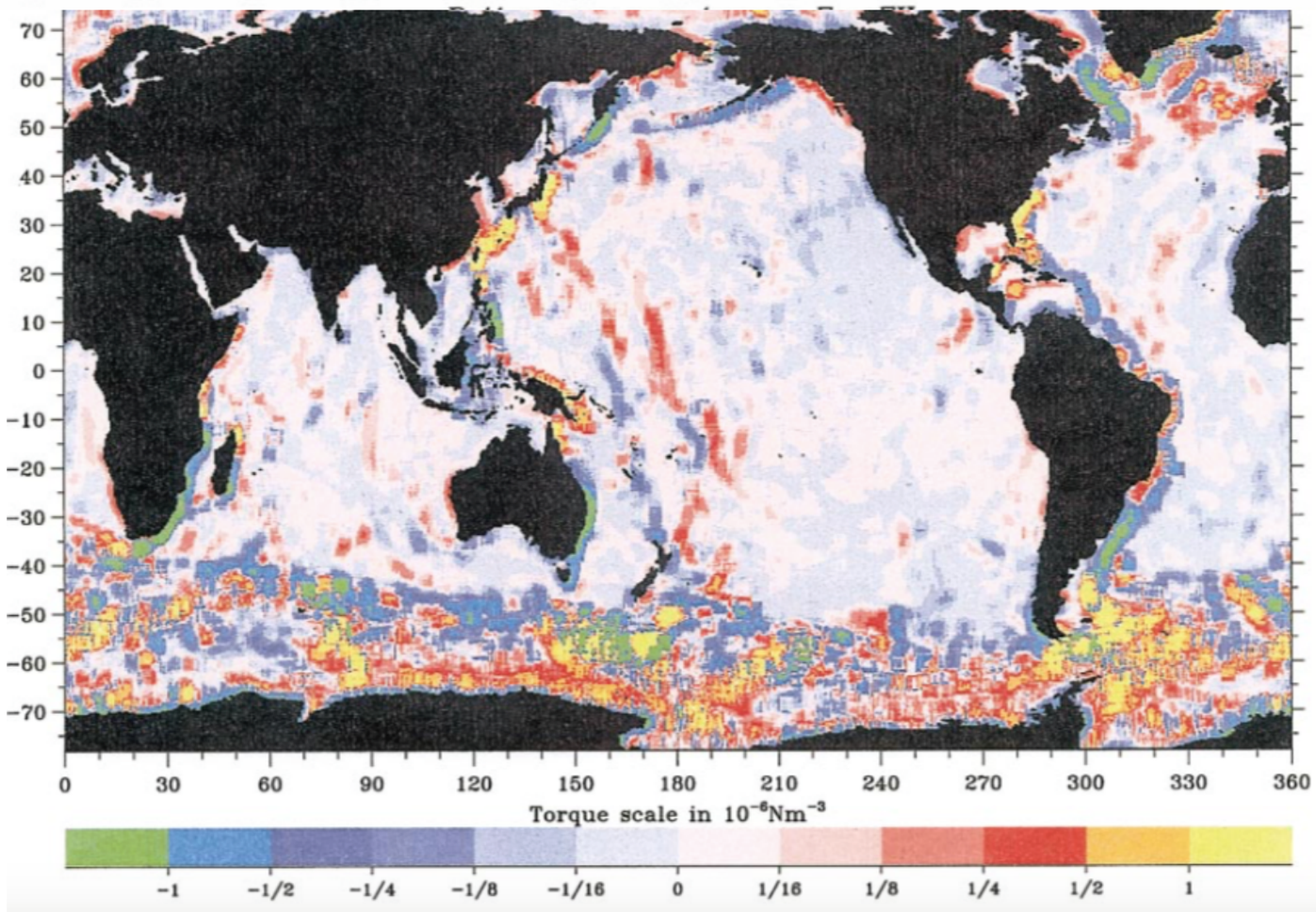
70
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Torque scale in 10⁻⁶Nm⁻³
−1
−1/2
−1/4
−1/8
−1/16
0
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1/8
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1

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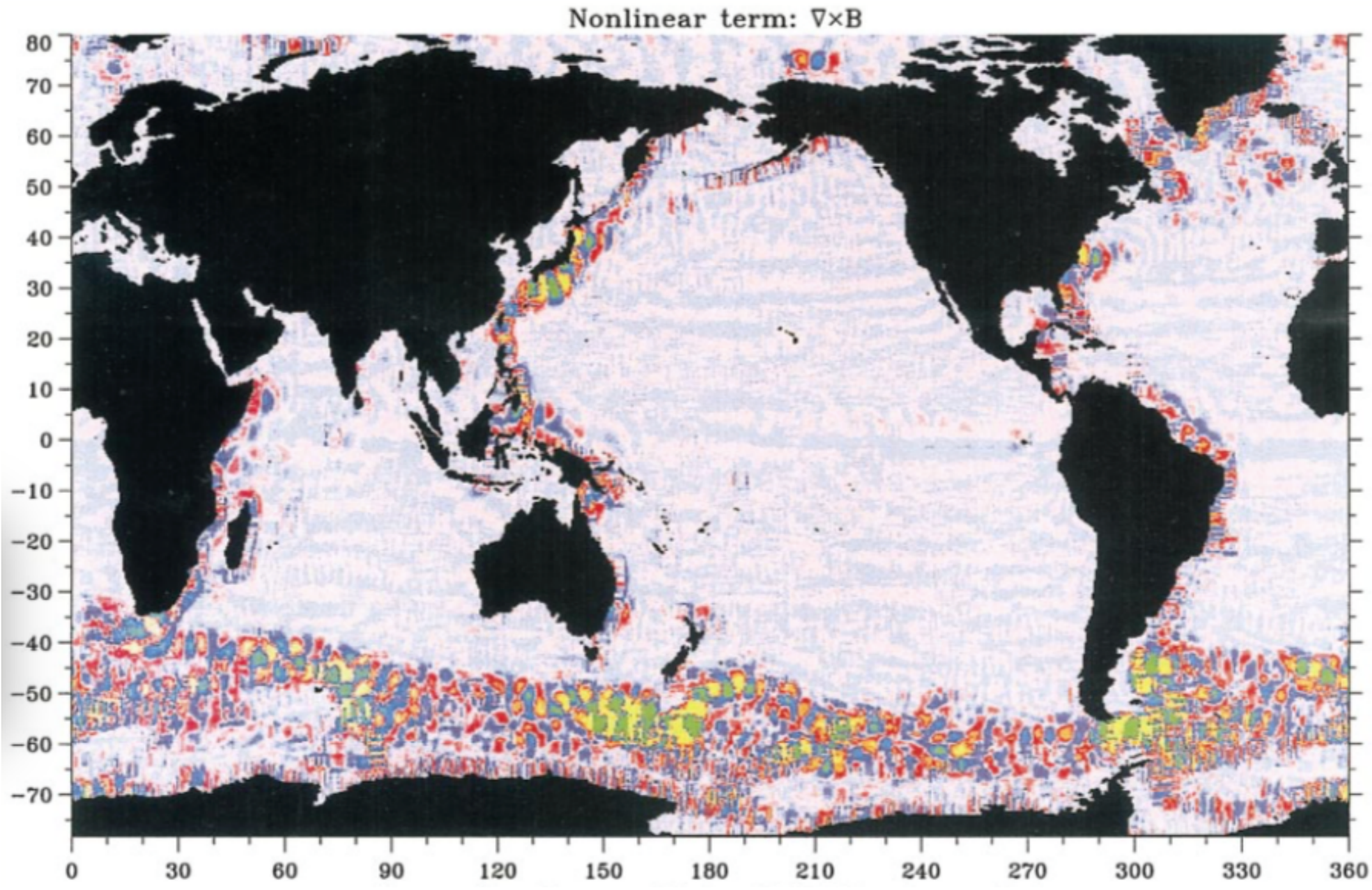

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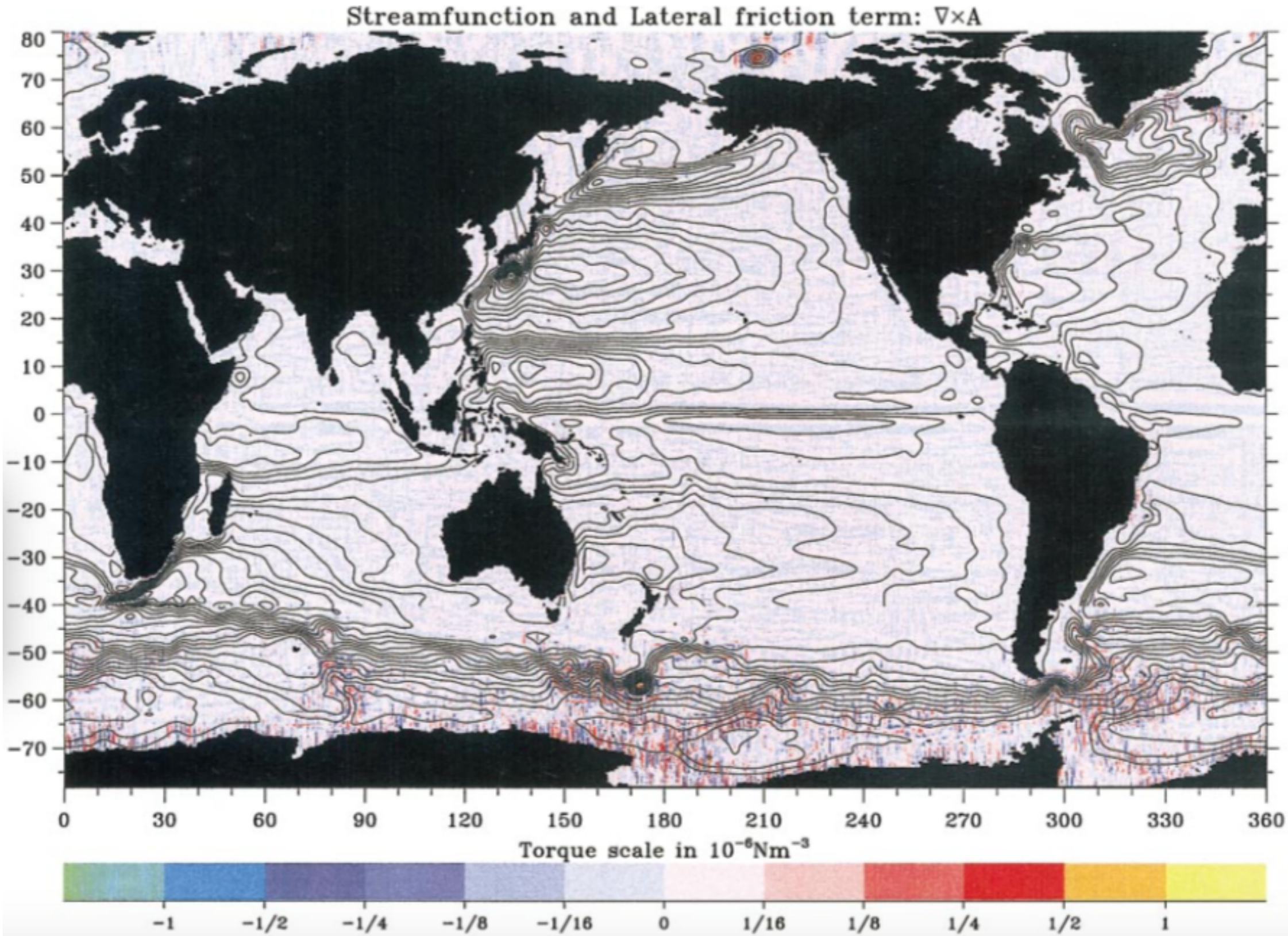
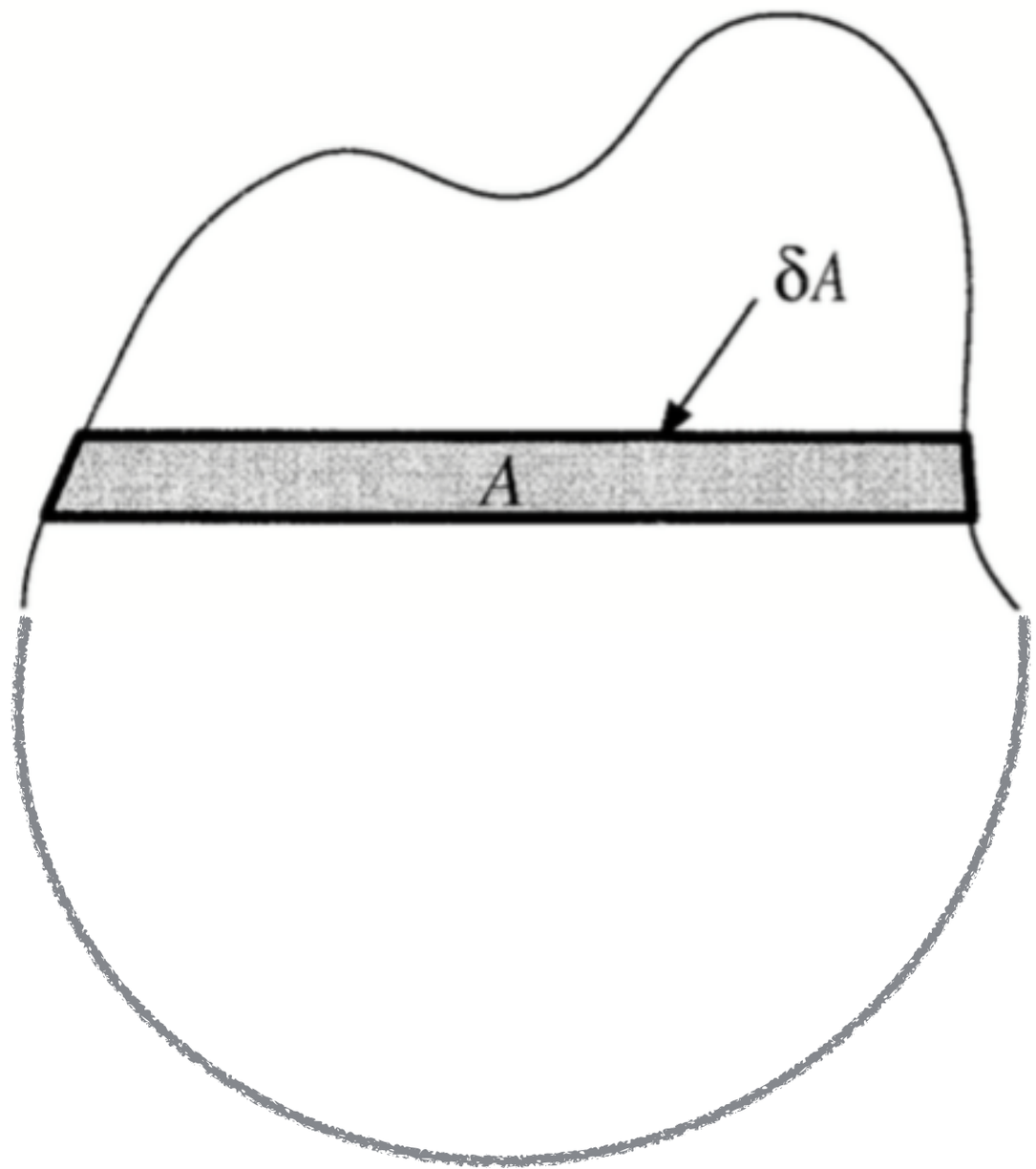
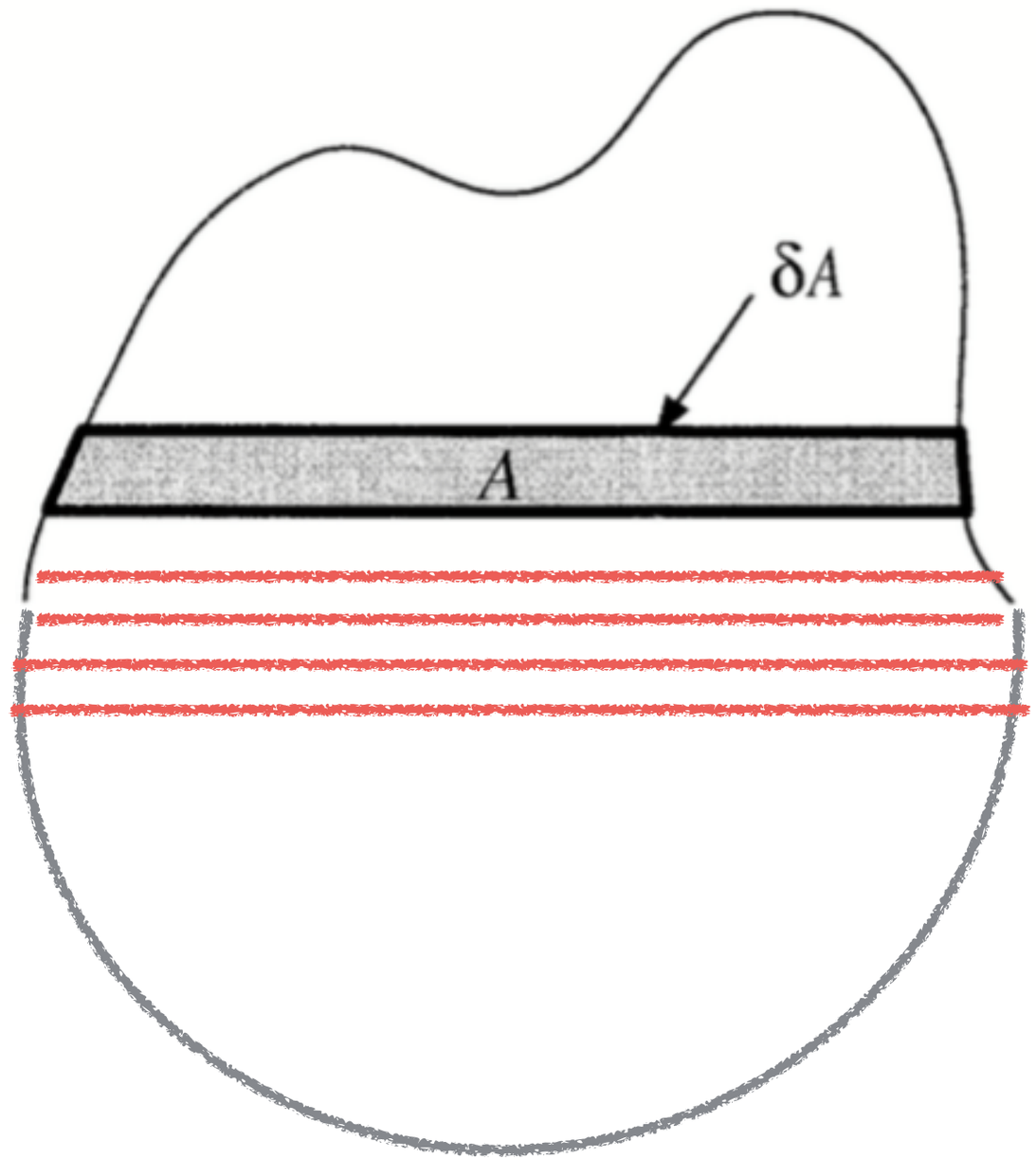
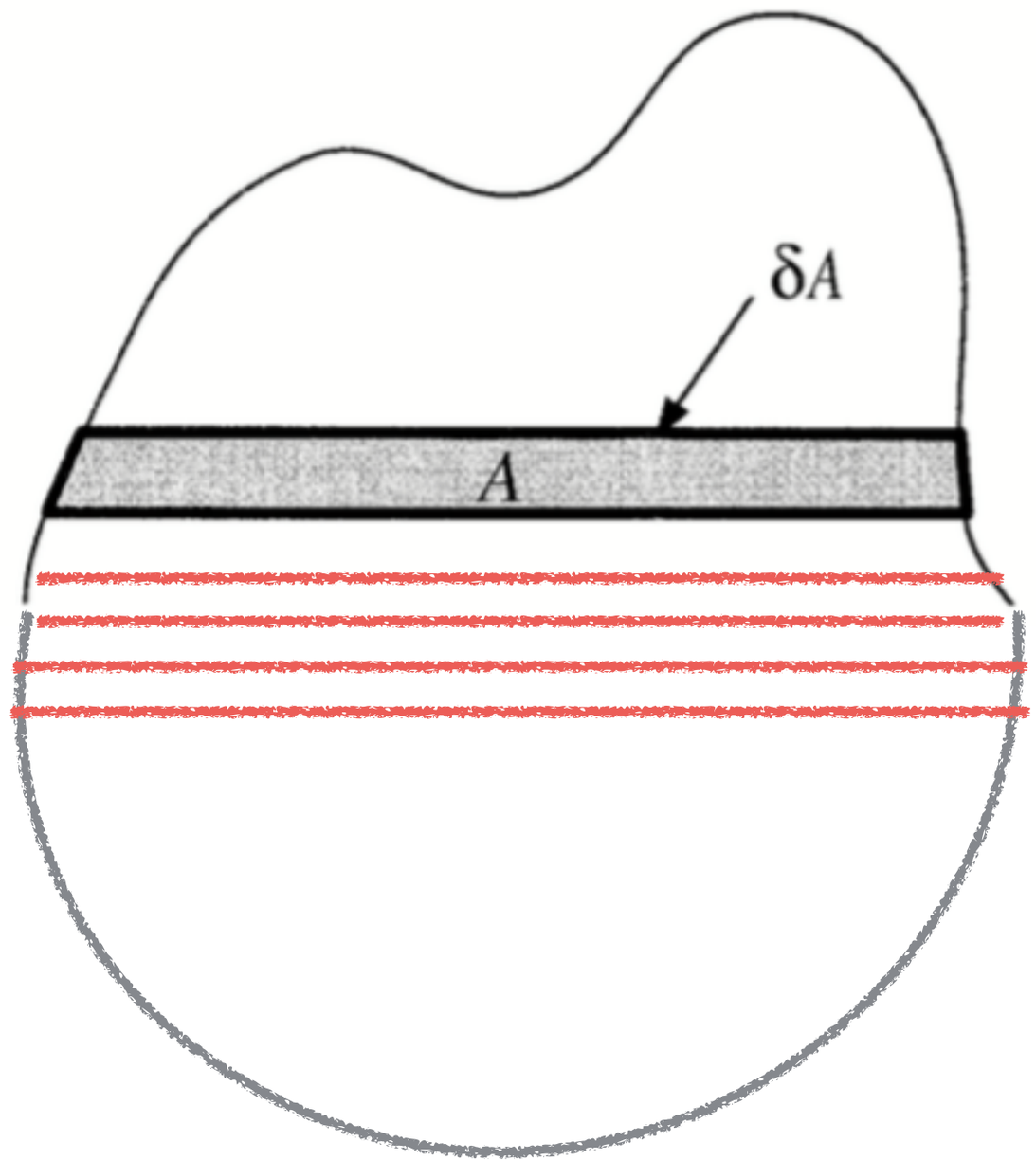


Another Look




δA
A

$$\sum A = \oint [f\hat{k} \times u = \nabla P + p_b \nabla H + \tau + Fric]$$



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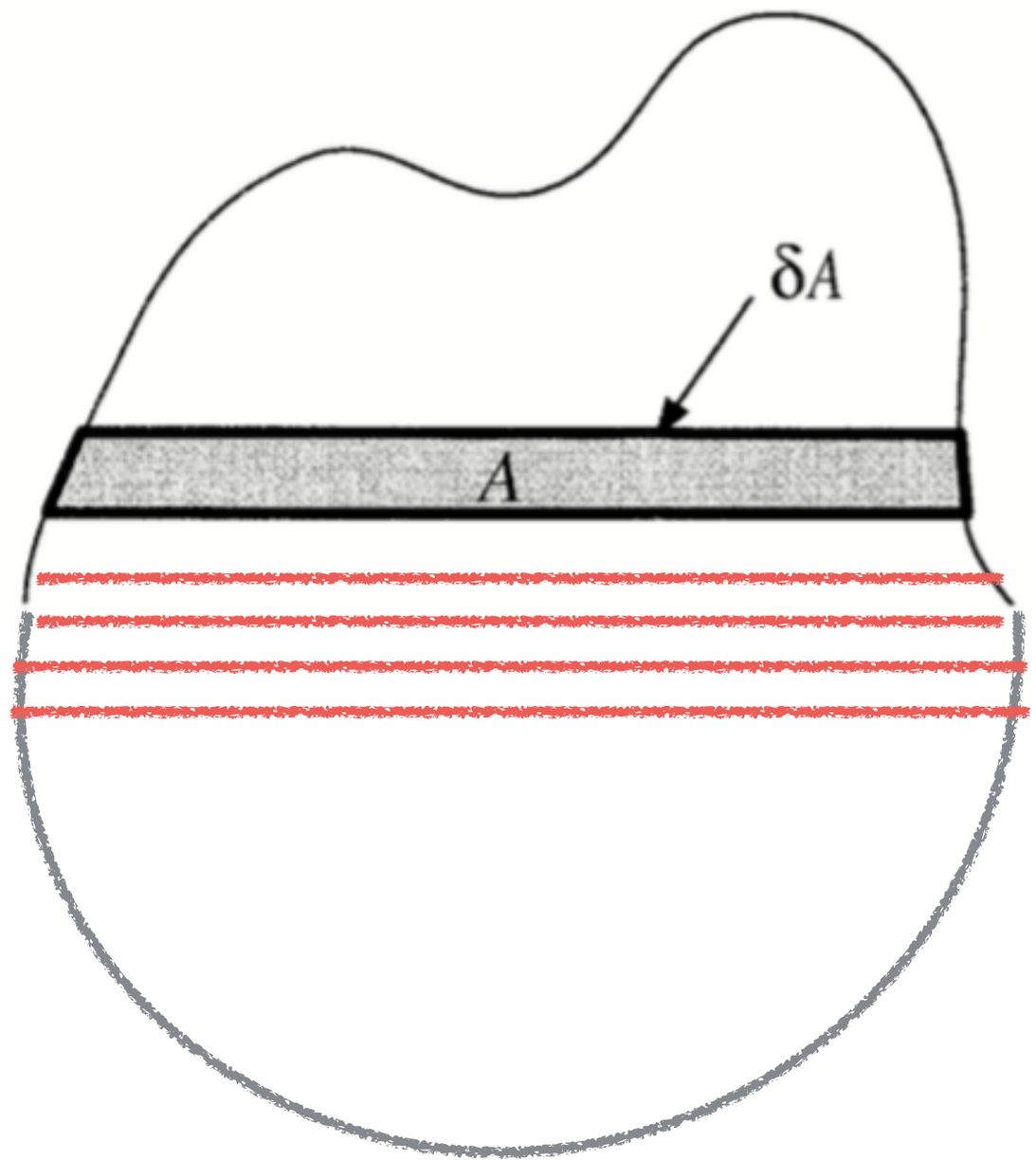

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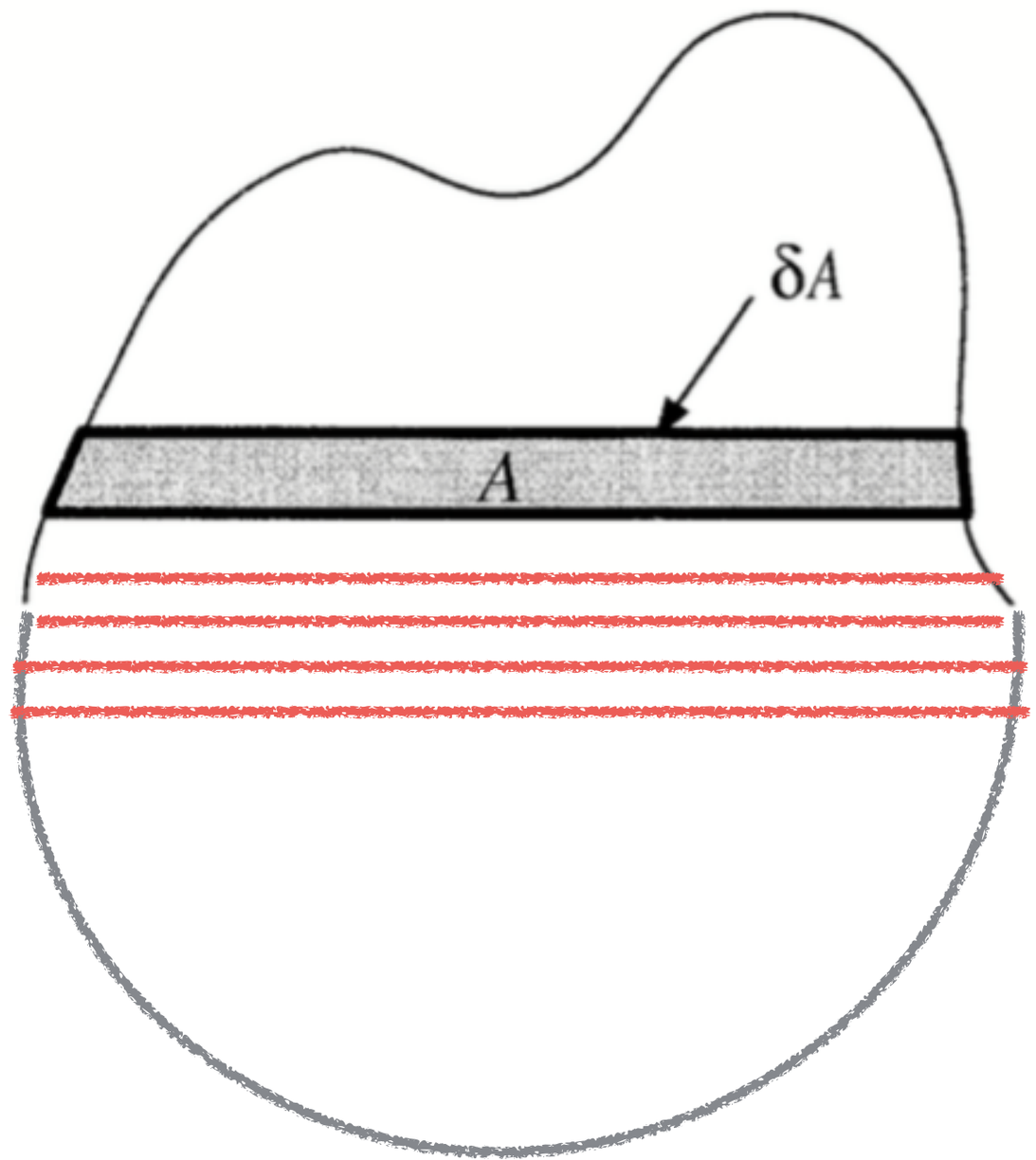

Discuss

Hence, it is only the alongshore wind stress that significantly upsets the interpretation of $\nabla \times \boldsymbol{\tau}_0$ as $\nabla \times \boldsymbol{\tau}_w$. Most analytical and idealized modeling studies deliberately choose to have no alongshore wind stress in order to avoid this problem. In reality there will be an alongshore wind stress that will result in a viscous boundary layer (as $\boldsymbol{\tau}_b$ increases from zero away from the coast to $\boldsymbol{\tau}_w$ at the coast, $\nabla \times \boldsymbol{\tau}_b$ must become significant in the boundary layer). This is clearly a separate issue from the question of whether western boundary currents are viscous since it occurs at all boundaries, east or west (or others), and the size of the effect is completely determined by the strength of the alongshore wind stress.

Although there is a balance between wind stress and form stress at each latitude to within about 10%, the relatively short length scale of variability in the nonlinear and form stress terms means that the gradients do not balance at each latitude, so the bottom pressure torque does not balance wind stress curl in a zonal integral at a single latitude. Typically, the latitude difference for which differences in zonal wind stress and form stress approximately balance is about three degrees of latitude, although a larger separation is necessary in parts of the Southern Ocean.