Topographic Enhancement of Eddy Efficiency in Baroclinic Equilibration

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by

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as told by Navid

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what's the problem?

what sets the thermocline depth *h*?
how topography affects it?

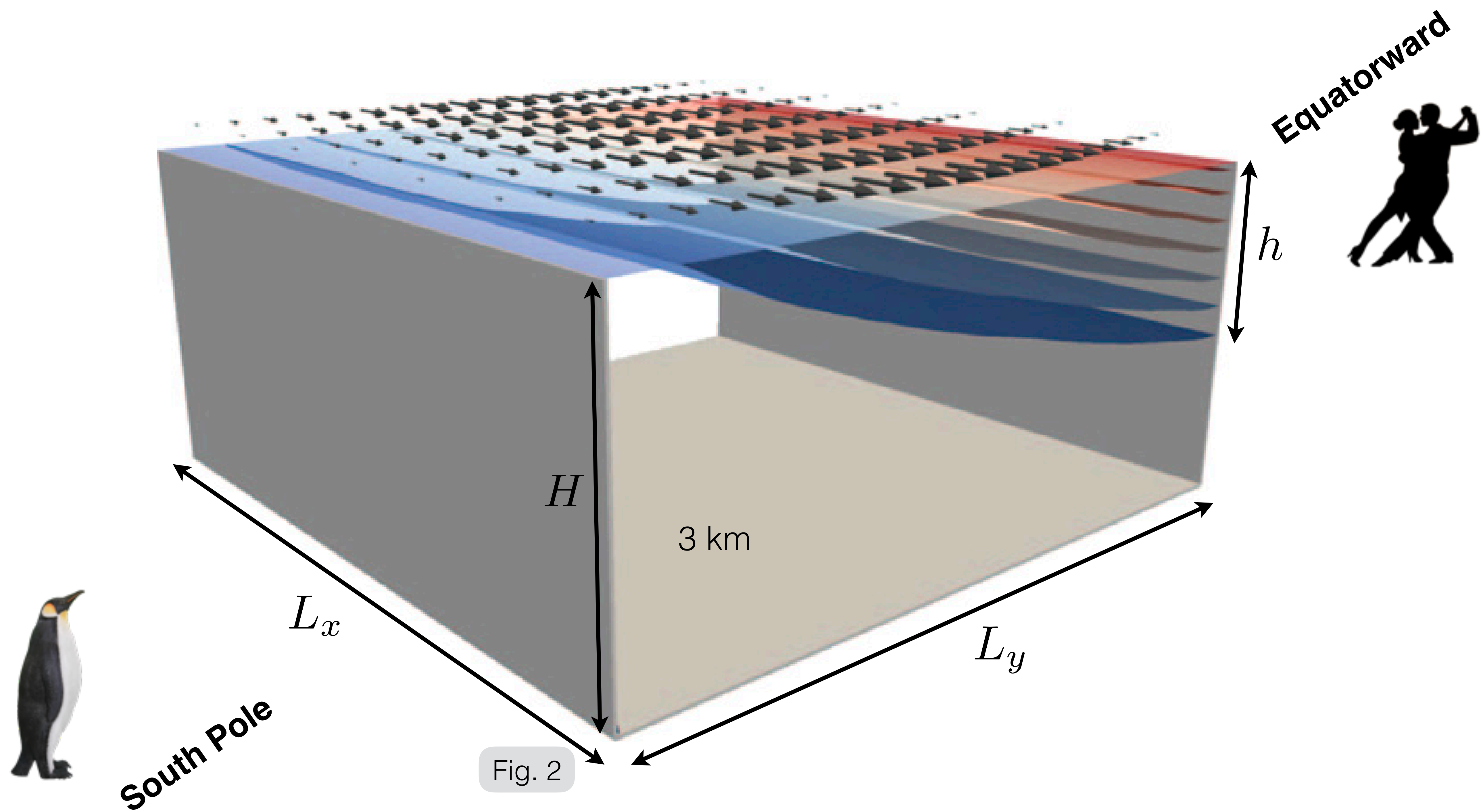


Fig. 2

section 2

topography makes thermocline shallower

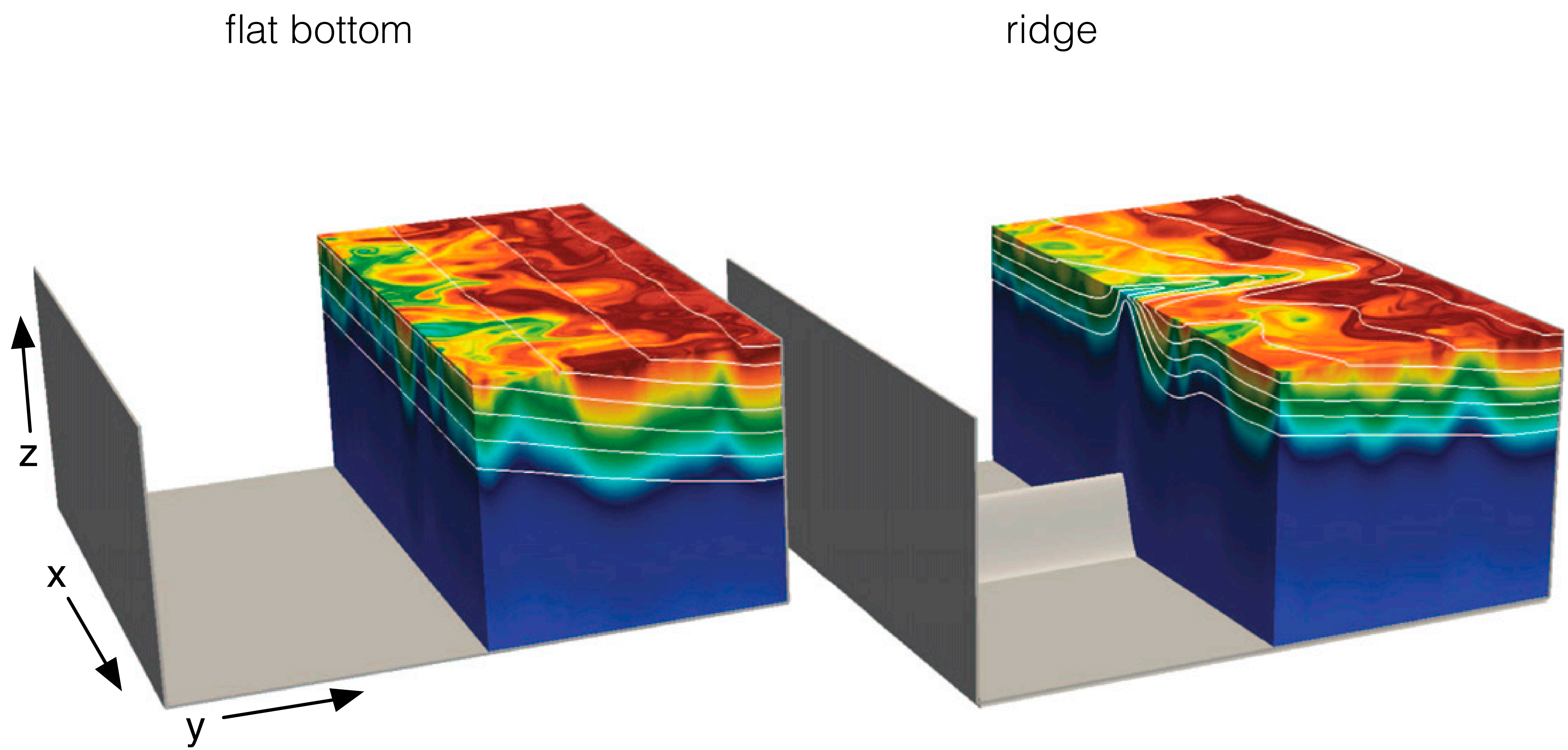


Fig. 4

isosurfaces of θ
colors from 0 ^oC to 8 ^oC
white lines: time-mean θ

section 2

meridional heat transport sets the thermocline depth

quasi-adiabatic
(no diapycnal mixing)

$$
\begin{aligned}
0 \approx \mathcal{H}(y) &= \rho_0 c_p L_x \int_{-H}^{0} \langle v(\theta - \theta_0) \rangle \, \mathrm{d}z \\
&= \mathcal{H}_{\text{mean}} \qquad + \qquad \mathcal{H}_{\text{eddy}} \\
&\approx -c_p L_x \frac{\tau}{f_0} \langle \theta \rangle + \rho_0 c_p L_x \int_{-H}^{0} \langle v_g \theta \rangle \, \mathrm{d}z \\
&= -c_p L_x \frac{\tau}{f_0} \Delta\theta y / L_y + \rho_0 c_p L_x h \, \widetilde{v_g \theta}(y)
\end{aligned}
$$

eddy efficiency
(title of paper)

$$\text{therefore near } y = L_y$$

$$h = \frac{\tau \, \Delta\theta}{f_0 \rho_0 \, \widetilde{v_g \theta}}$$

the goal is to understand how topography affects the *efficiency*

model setup

MITgcm, hydrostatic Boussinesq eqs. on a β-plane

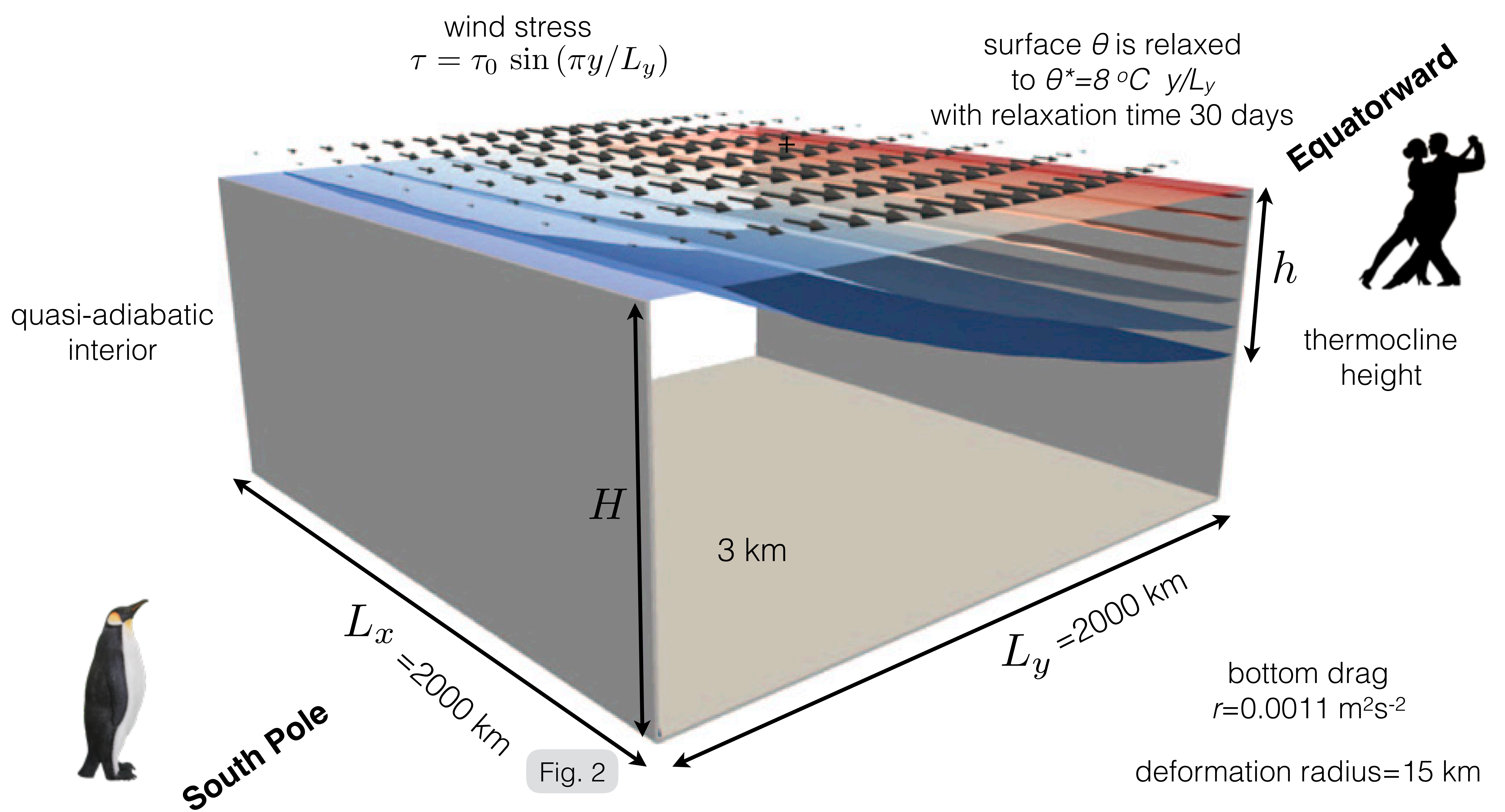


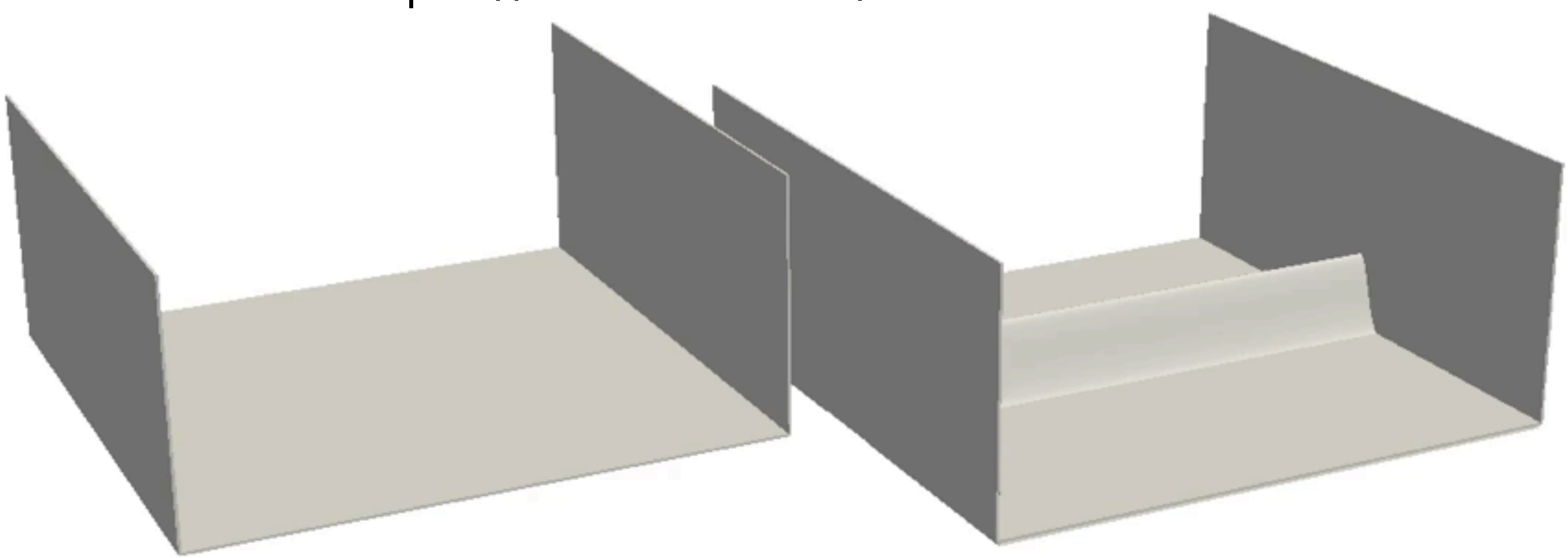
Fig. 2

model spinup for τ_0=0.2 N/m^2

flat bottom

ridge

https://vimeo.com/55486114



equilibration
~100 yr

isosurfaces of θ
colors from 0 ^oC to 8 ^oC

section 3

fields decomposition

MEAN	Standing Wave (SE)	Transient (TE)

$$A \;=\; \langle A\rangle(y,z) \;+\; A^{\dagger}(x,y,z) \;+\; A'(x,y,z,t)$$

time mean of the zonal mean	time mean of the deviation from the zonal mean	the rest…

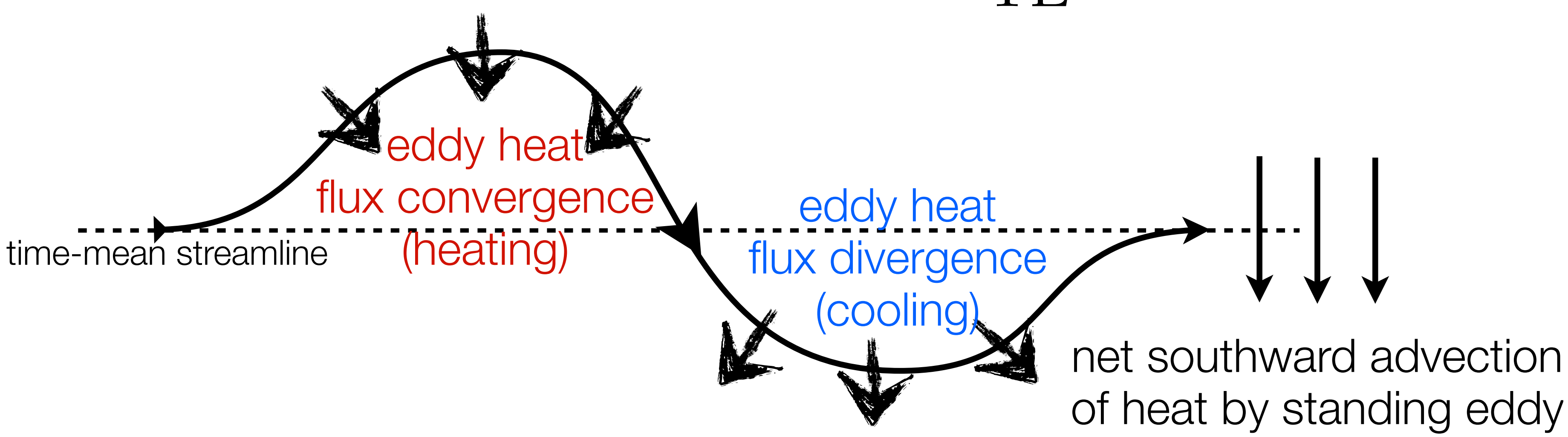
$$\int A^{\dagger}\,\mathrm{d}x = 0 \qquad \int A'\,\mathrm{d}t = 0$$

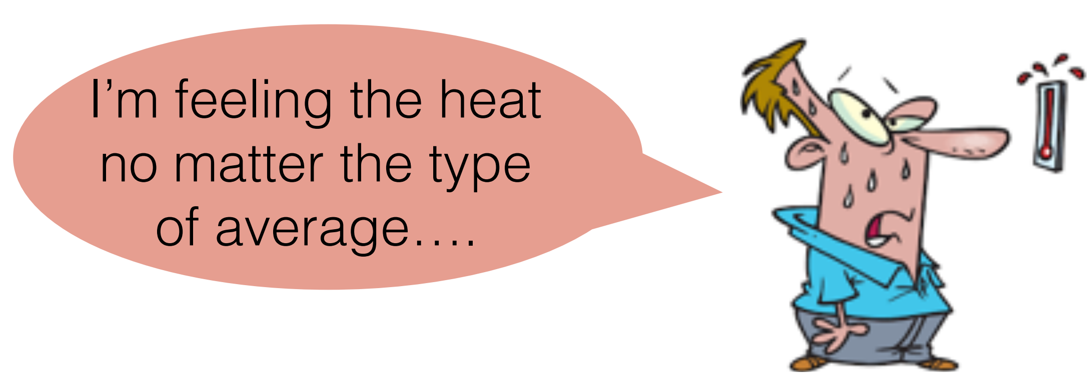
section 3

averages over latitude circles Vs over streamlines

$$\mathcal{H}(y) = \rho_0 c_p L_x \int_{-H}^{0} \langle v(\theta - \theta_0) \rangle \, \mathrm{d}z$$
$$= \mathcal{H}_{\mathrm{mean}} + \mathcal{H}_{\mathrm{SE}} + \mathcal{H}_{\mathrm{TE}}$$
$$= \mathcal{H}^{\Theta}_{\mathrm{mean}} + \mathcal{H}^{\Theta}_{\mathrm{TE}}$$

$$\mathcal{H}_{\mathrm{SE}} + \mathcal{H}_{\mathrm{TE}} \approx \mathcal{H}^{\Theta}_{\mathrm{TE}}$$





section 3

topography makes the thermocline shallower

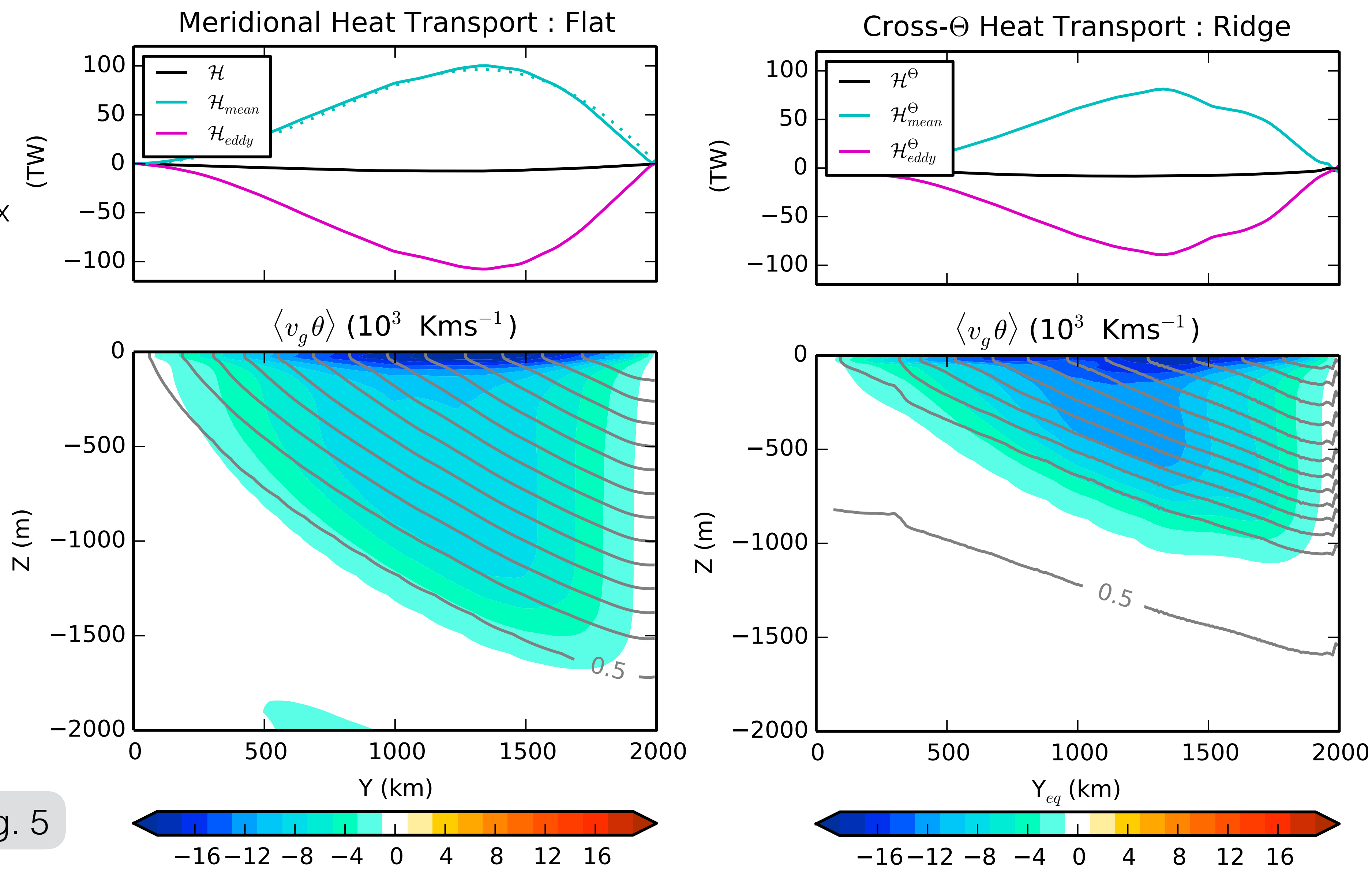
meridional heat transport: $$\mathcal{H}(y) = \rho_0 c_p L_x \int_{-H}^{0} \langle v(\theta - \theta_0) \rangle \,\mathrm{d}z$$

$$= \mathcal{H}_{\mathrm{mean}} + \mathcal{H}_{\mathrm{SE}} + \mathcal{H}_{\mathrm{TE}}$$ (average over lat. circles)

$$= \mathcal{H}^{\Theta}_{\mathrm{mean}} + \mathcal{H}^{\Theta}_{\mathrm{TE}}$$ (average over streamlines)

(the mean is very close to the Ekman flux calculation)

h=1200 m



h=1000 m

higher *eddy efficiency*

Fig. 5

flat bottom

ridge

flat Vs ridge results I

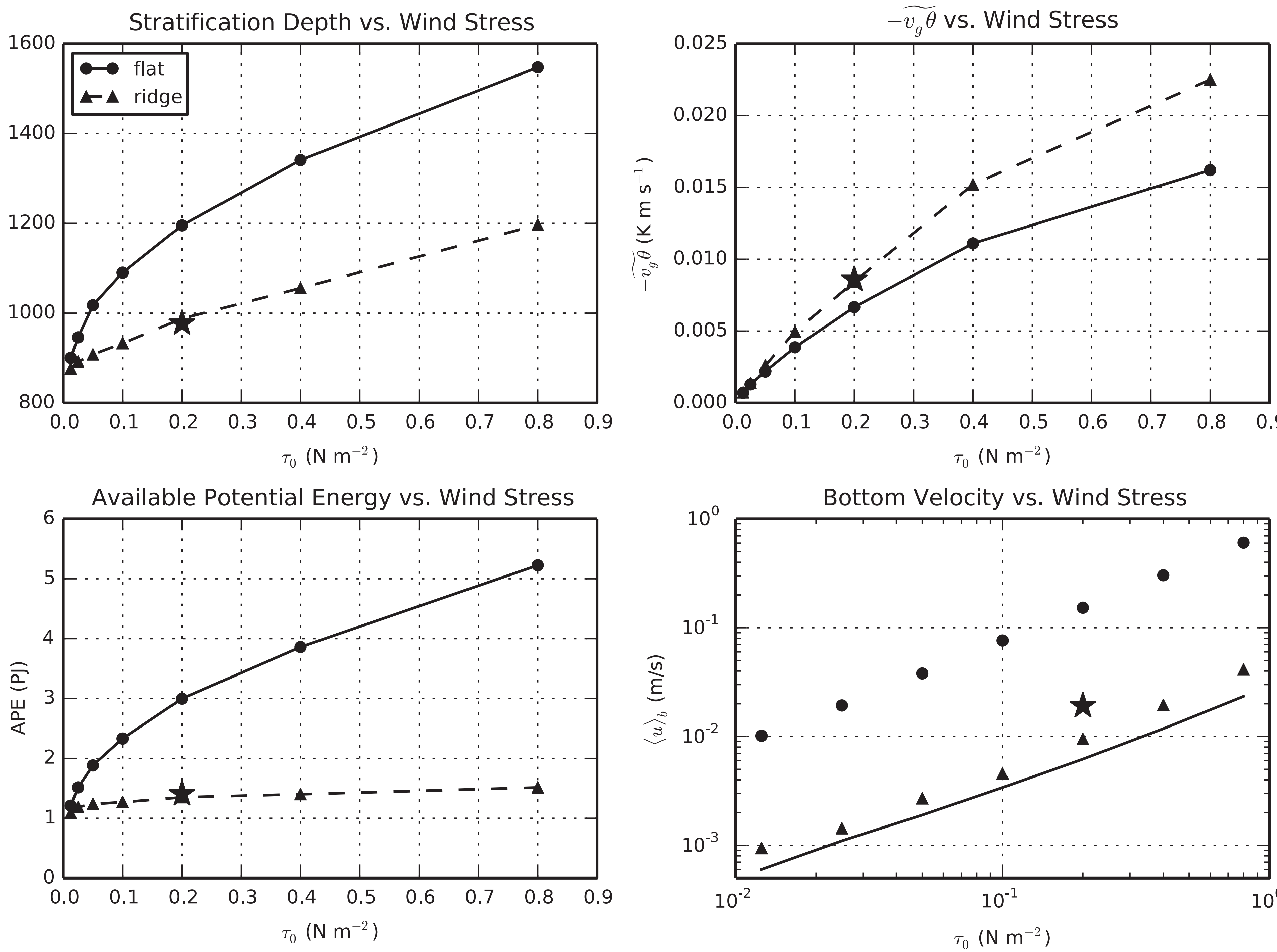


Fig. 6

(Munk & Palmen)

flat Vs ridge results II

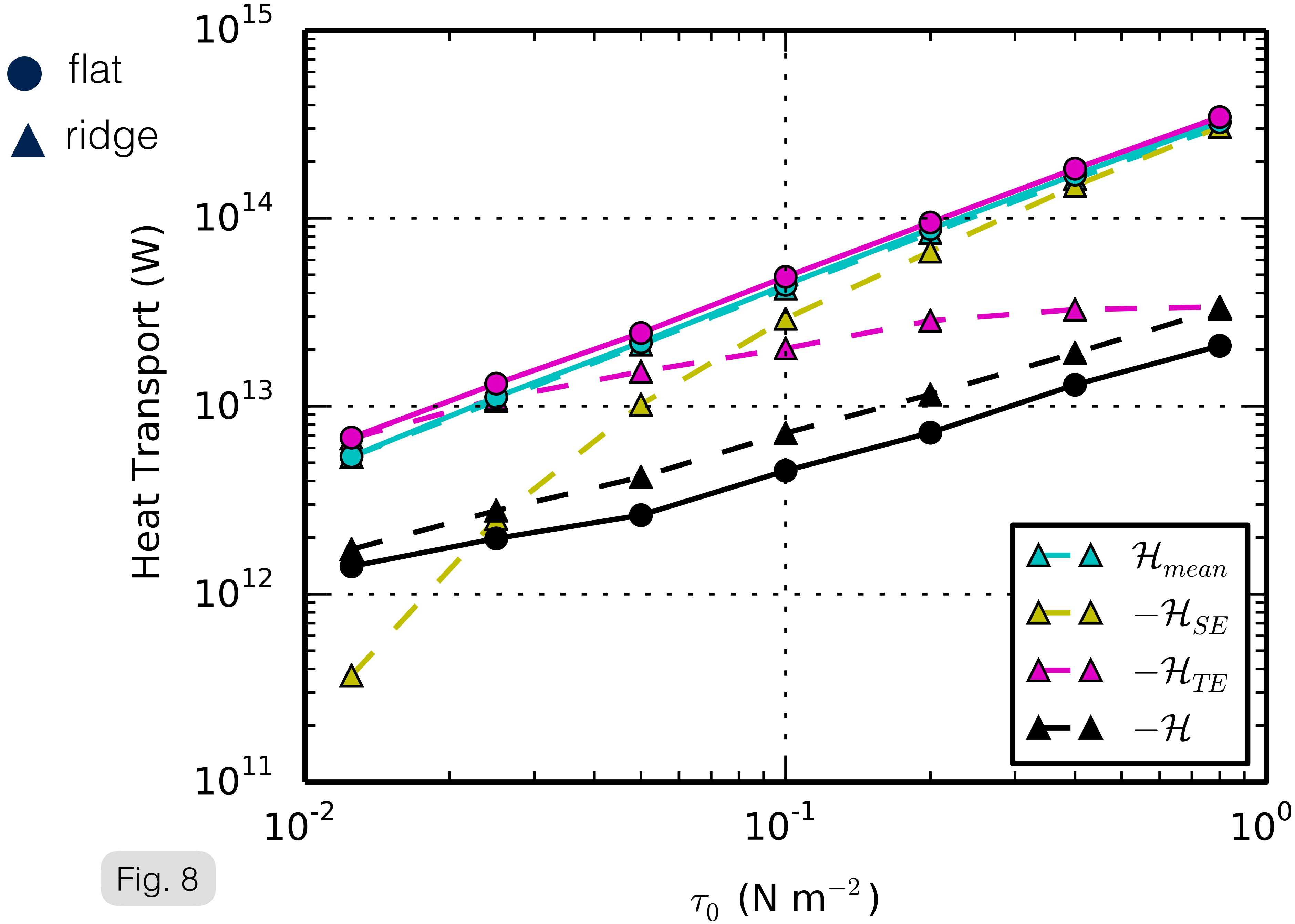
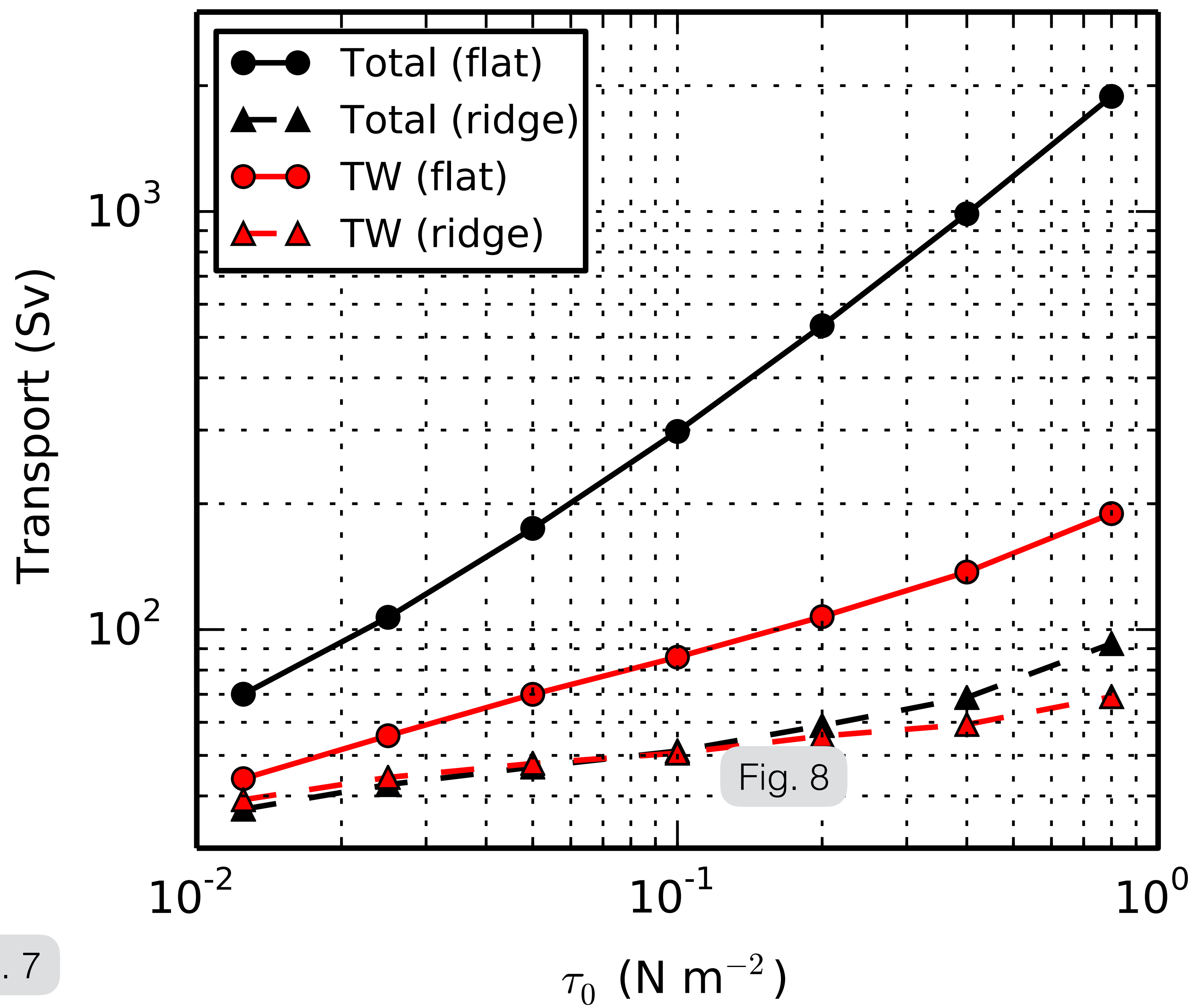


Fig. 8

when there is topography as wind increases the heat flux
is mainly be done by the SE rather than the TE

flat Vs ridge results III

flat
ridge



(Munk & Palmen)

Fig. 7

$$\text{transport} = \int_0^{L_y} \mathrm{d}y \int_{-H}^{0} \mathrm{d}z \, \langle u \rangle \approx \int_0^{L_y} \mathrm{d}y \langle u_{\text{bottom}} \rangle + \underbrace{\frac{ga}{f_0} \int_0^{L_y} \mathrm{d}y \int_{-H}^{0} \mathrm{d}z \, \frac{\partial \langle \theta \rangle}{\partial y} z}_{\text{TW}}$$

a simplified 2-layer QG model

PV at each layer

$$q_1 = \nabla^2 \psi_1 + F_1(\psi_2 - \psi_1)$$

$$q_2 = \nabla^2 \psi_2 + F_2(\psi_1 - \psi_2) \; + \frac{f_0}{H_2} h_b$$

decompose the flow fields into:

$$\psi_j(x, y, t) \; = \; \langle \psi_j \rangle (y) \; + \; \psi_j^{\dagger}(x, y) \; + \; \psi'(x, y, t) \qquad j = 1, 2$$

MEAN | Standing Wave (SE) | Transient (TE)

a simplified 2-layer QG model

we get then an equation for the TE

$$(\partial_t + U_1\partial_x)(q_1^\dagger + q_1') + \mathsf{J}(\psi_1^\dagger + \psi_1', q_1^\dagger + q_1') + \partial_y\langle q_1\rangle\partial_x(\psi_1^\dagger + \psi_1') = -\frac{\partial_y\tau}{\rho_0 H_1}$$

$$(\partial_t + U_2\partial_x)(q_2^\dagger + q_2') + \mathsf{J}(\psi_2^\dagger + \psi_2', q_2^\dagger + q_2') + \partial_y\langle q_2\rangle\partial_x(\psi_2^\dagger + \psi_2') = -\frac{r}{H_2}\left[\nabla^2(\psi_2^\dagger + \psi_2') - \partial_y U_2\right]$$

which if we time average gives us the
equations for the standing wave component

$$U_1\partial_x q_1^\dagger + \partial_y\langle q_1\rangle\partial_x\psi_1^\dagger + \mathsf{J}(\psi_1^\dagger, q_1^\dagger) + \overline{\mathsf{J}(\psi_1', q_1')}$$
$$- \partial_y\langle(\partial_x\psi_1^\dagger)q_1^\dagger\rangle - \partial_y\langle(\partial_x\psi_1')q_1'\rangle = 0$$

where did the
wind stress go?

$$U_2\partial_x q_2^\dagger + \partial_y\langle q_2\rangle\partial_x\psi_2^\dagger + \mathsf{J}(\psi_2^\dagger, q_2^\dagger) + \overline{\mathsf{J}(\psi_2', q_2')}$$
$$- \partial_y\langle(\partial_x\psi_2^\dagger)q_2^\dagger\rangle - \partial_y\langle(\partial_x\psi_2')q_2'\rangle = -\frac{r}{H_2}\nabla^2\psi_2^\dagger$$

a simplified 2-layer QG model

$$\partial_x \psi^\dagger \sim U \quad , \quad \partial_y \psi^\dagger \ll U \implies \text{neglect } \partial_y$$

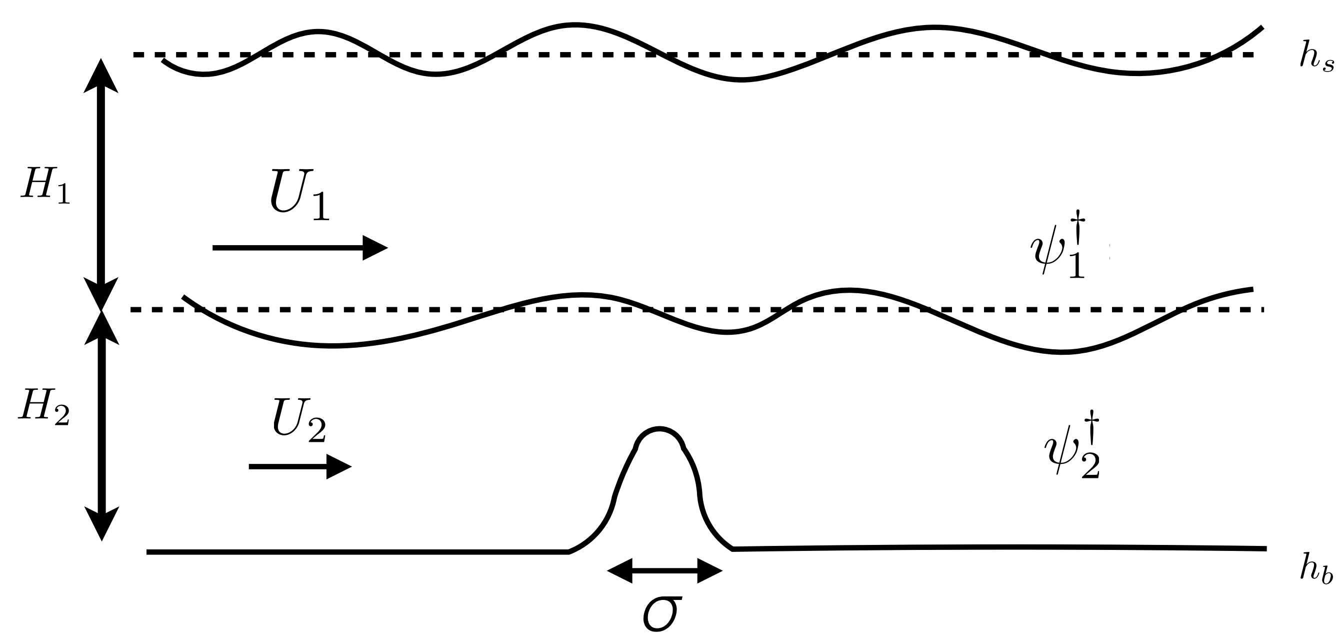
$$\text{neglect terms quadratic in } \psi^\dagger$$

$$\text{parametrize } \overline{\mathsf{J}(\psi_j', q_j')} = -K\,\nabla^2 \overline{q_j}$$

$$U_1 \partial_x^2 \psi_1^\dagger + U_1 F_1 \psi_2^\dagger + \beta \psi_1^\dagger - F_1 U_2 \psi_1^\dagger = K \partial_x q_1^\dagger$$

Eqs (30)-(31)

$$U_2 \partial_x^2 \psi_2^\dagger + U_2 F_2 \psi_1^\dagger + \beta \psi_2^\dagger - F_2 U_1 \psi_2^\dagger = K \partial_x q_2^\dagger - \frac{r}{H_2} \partial_x \psi_2^\dagger - \frac{f_0}{H_2} U_2 h_b$$



the standing wave components

$$U_1\partial_x^2\psi_1^\dagger + U_1F_1\psi_2^\dagger + \beta\psi_1^\dagger - F_1U_2\psi_1^\dagger = K\partial_x q_1^\dagger$$

$$U_2\partial_x^2\psi_2^\dagger + U_2F_2\psi_1^\dagger + \beta\psi_2^\dagger - F_2U_1\psi_2^\dagger = K\partial_x q_2^\dagger - \frac{r}{H_2}\partial_x\psi_2^\dagger - \frac{f_0}{H_2}U_2h_b$$

(ridge>deformation) $\sigma \gg \dfrac{1}{\sqrt{F_2}}$

(K is small
or
TE effect negligible) $\dfrac{K}{\sigma U_2} \ll 1$ $\longrightarrow$ $U_2\psi_1^\dagger \approx U_1\psi_2^\dagger$

(Rhines scale = ridge) $\sqrt{\dfrac{U_1}{\beta}} \approx \sigma$

using this approximation one
can proceed to calculate …

section 4

… the heat transport in the QG model

$$\mathcal{H}_{\mathrm{mean}} + \boxed{\mathcal{H}_{\mathrm{SE}} + \mathcal{H}_{\mathrm{TE}}}$$

$$\langle \psi_1^\dagger \partial_x \psi_2^\dagger \rangle + \langle \psi_1' \partial_x \psi_2' \rangle = K (U_1 - U_2) \left(1 + \frac{\langle (\partial_x \psi_2^\dagger)^2 \rangle}{U_2^2} \right)$$

eddy heat transport is augmented by the presence of the standing wave ψ_2 due to the topography

… and the thermocline slope in the QG model

$$s = f_0 \frac{U_1 - U_2}{g'} = \frac{\tau_0/(\rho_0 K f_0)}{1 + \frac{\langle (\partial_x \psi_2^\dagger)^2 \rangle}{U_2^2}}$$

the planetary scale slope isopycnal is reduced
due to the standing wave ψ_2

the QG model captures the qualitative behavior

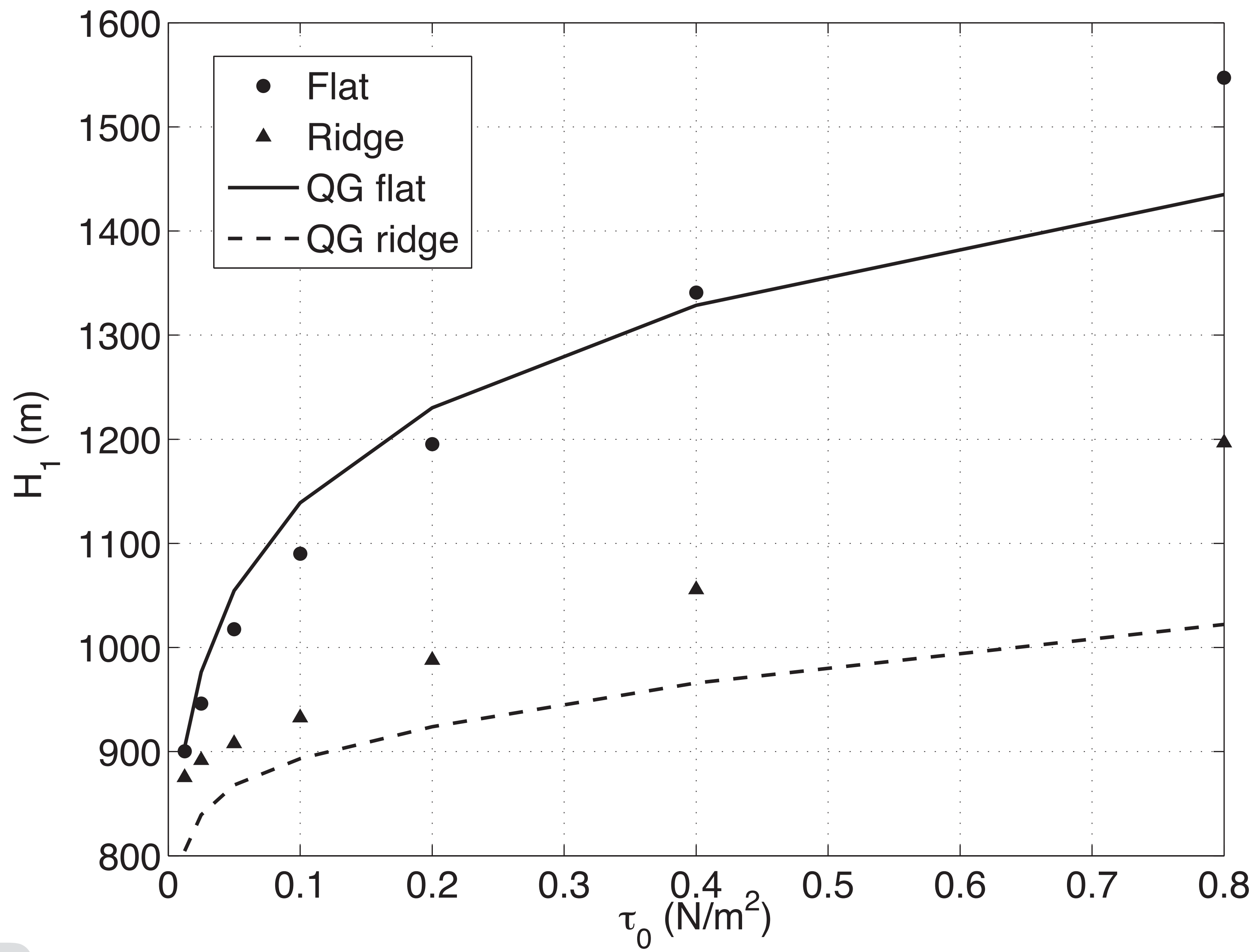


Fig. 9

section 5

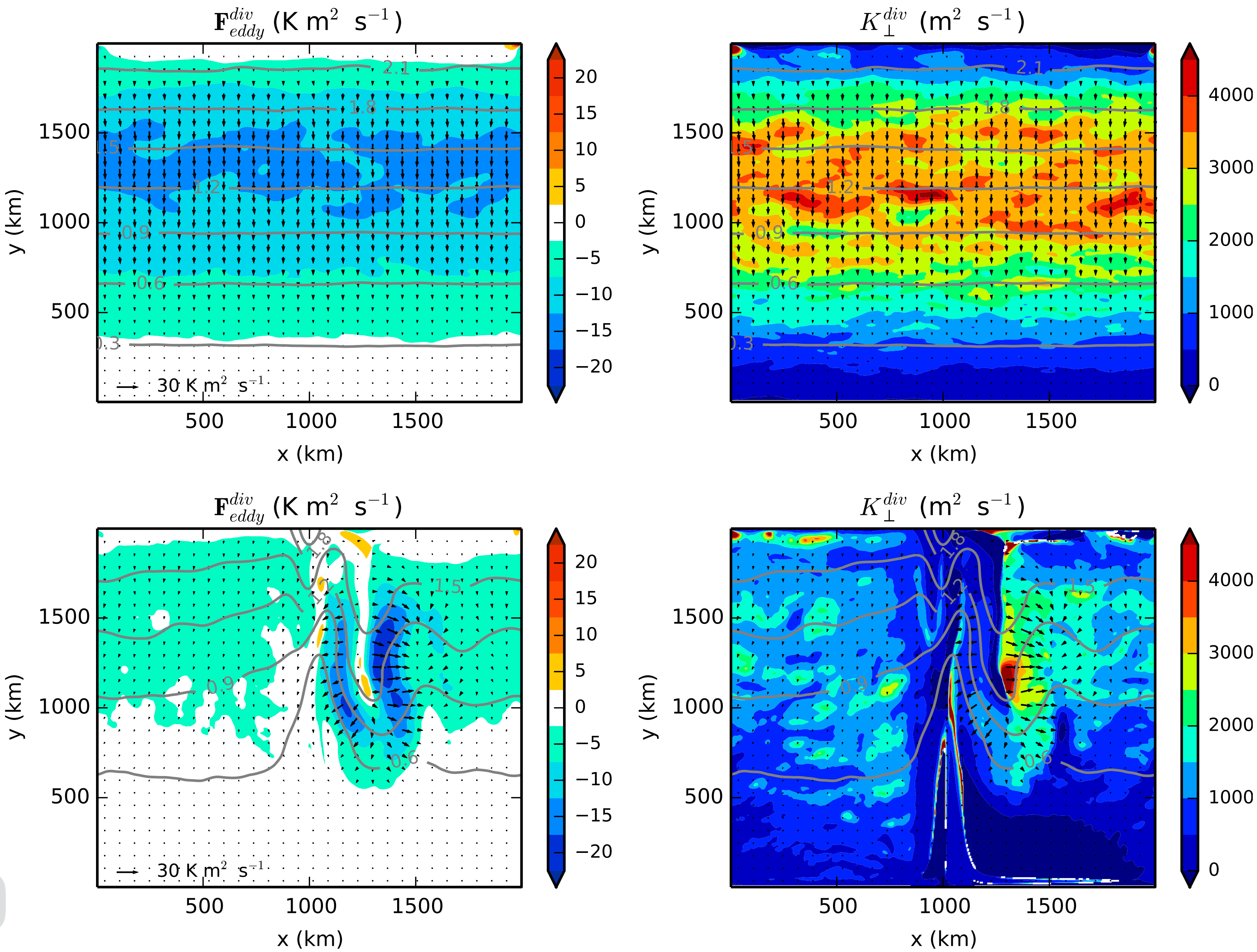
local cross-stream heat fluxes

$$\mathcal{H}^{\Theta_0} = \rho_0 c_p \int_{\Theta<\Theta_0} \nabla \cdot \mathbf{F}\, \mathrm{d}x\, \mathrm{d}y \quad , \qquad \mathbf{F} = \int_{-H}^{0} \overline{\mathbf{u}\theta}\, \mathrm{d}z = \int_{-H}^{0} \left(\overline{\mathbf{u}}\overline{\theta} + \overline{\mathbf{u}'\theta'}\right) \mathrm{d}z$$

$$= \mathbf{F}_{\mathrm{mean}} + \mathbf{F}_{\mathrm{eddy}} \longrightarrow \mathbf{F}^{\mathrm{div}}_{\mathrm{mean}} + \mathbf{F}^{\mathrm{div}}_{\mathrm{eddy}}$$

flat bottom

ridge



the ridge enhances diffusivity near it but suppresses it overall resulting in *smaller* mean diffusivity

Fig. 11

how fast the eddies are carried by the flow?

flat bottom

ridge

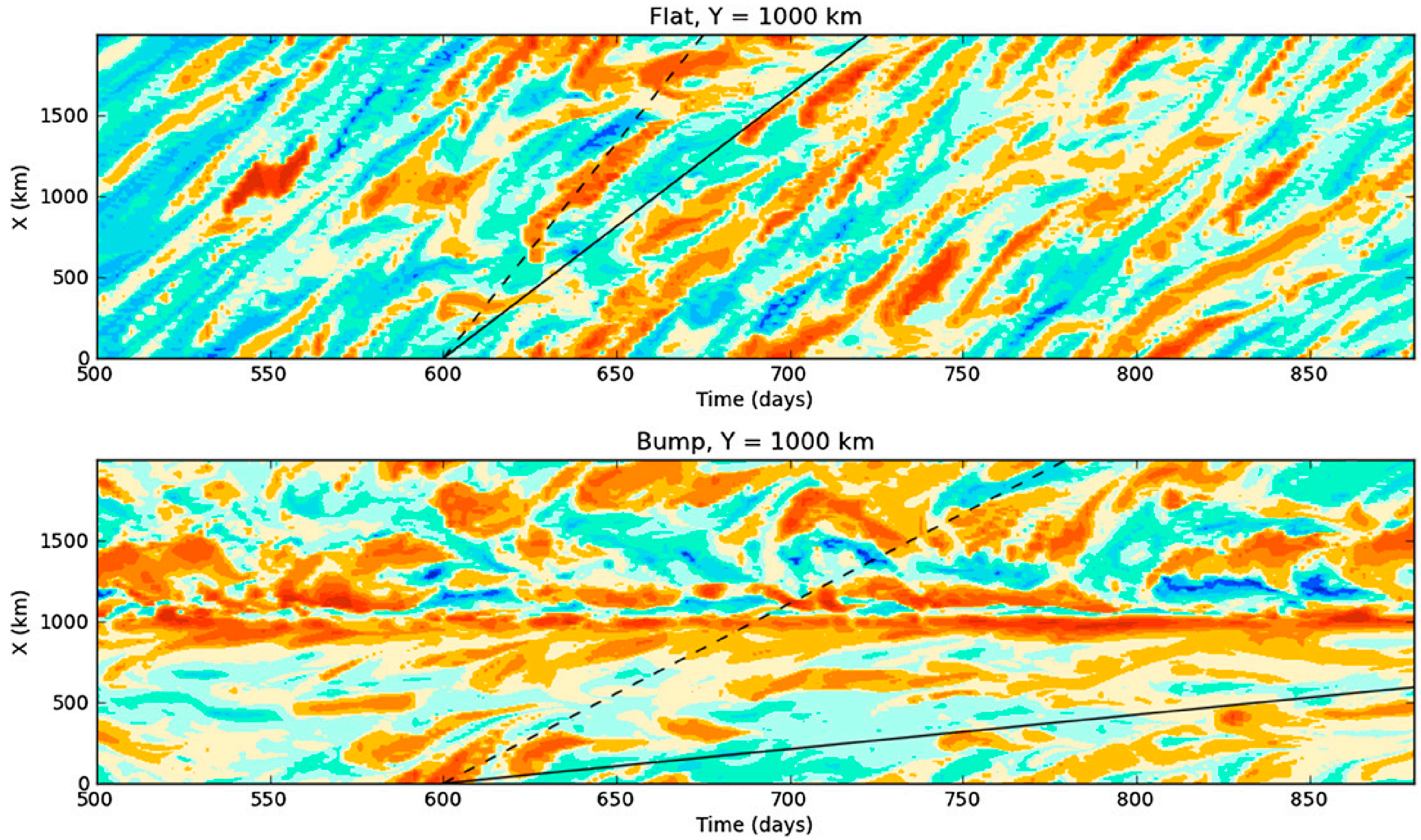


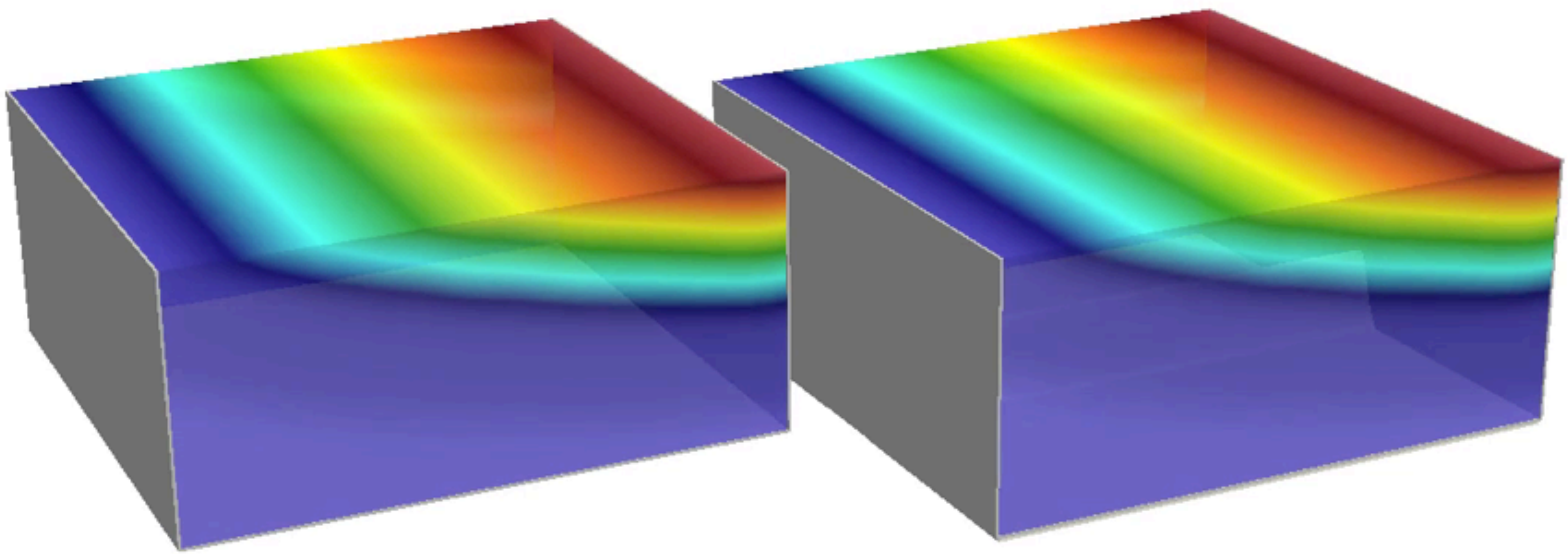
Fig. 13

eddies propagate much faster without topography

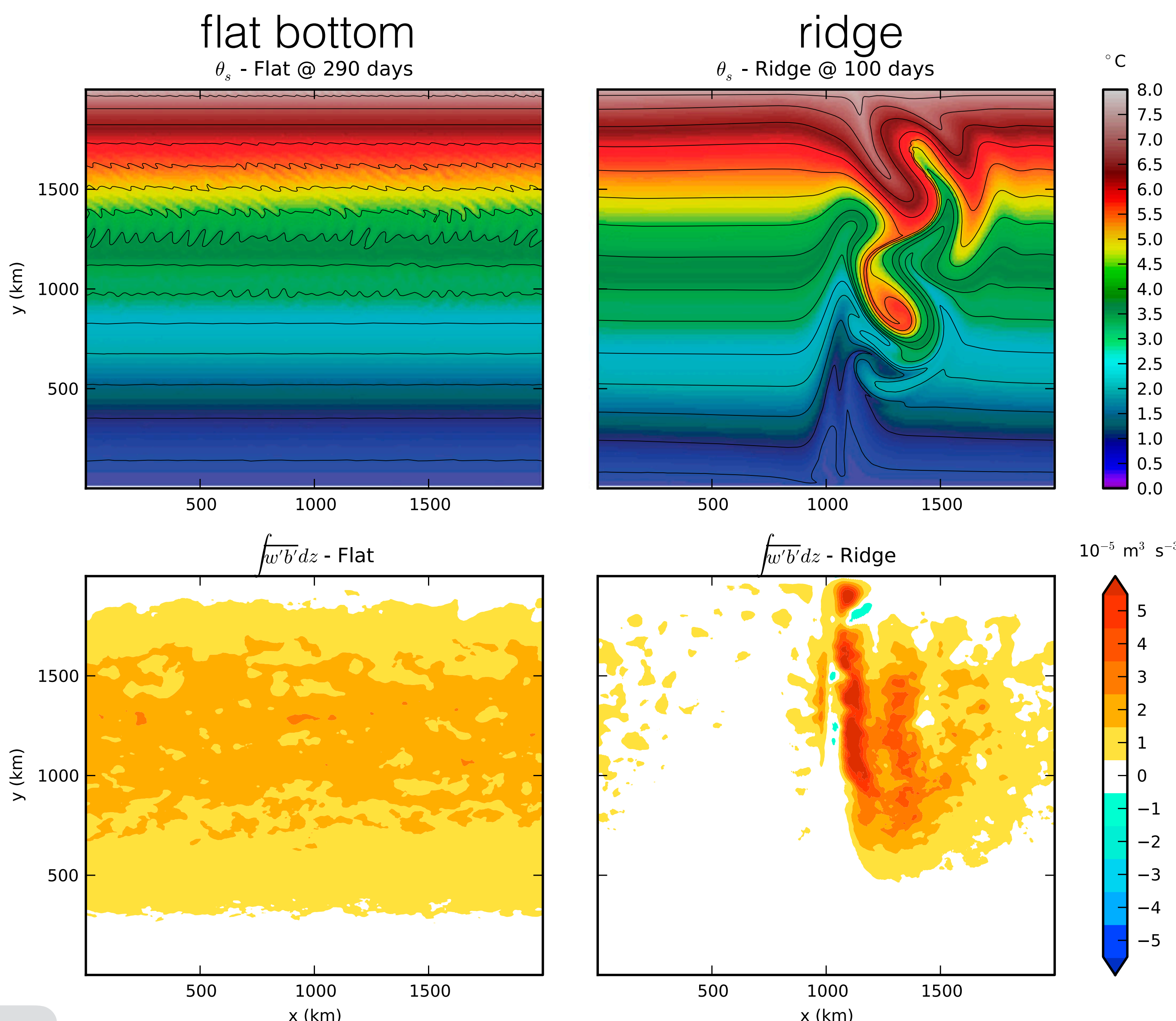
authors argue that eddy production is done through *convective instability* for the flat case
and
through *absolute instability* for the ridge case

convective Vs absolute instability

see https://vimeo.com/55486114



local cross-stream heat fluxes



conversion from PE to KE

Fig. 14

EKE is produced mainly downstream the ridge

discussion

- We never clearly see the SE in the full model (only its signature in the Hovmöller diagram)…
- Why $\mathcal{H}_{\text{mean}} \approx \mathcal{H}_{\text{Ekman}}$?
- $\mathcal{H}_{\text{SE}} \gg \mathcal{H}_{\text{TE}}$ as wind stress increases. How this translates to the averaging-over-streamlines picture?
- "*The inability of existing parameterizations to account for local instability and nonlocal eddy life cycles constitutes the main obstacle toward a more complete theory of baroclinic equilibration in the presence of large topography and the more general problem of inhomogenous geostrophic turbulence.*"