Contours of slopes of a rippled water surface

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Abstract: The appearance of a horizontal array of linear lamps below the water surface when viewed from above is approximately in the form of contours of one component of the water surface slope. The degree of approximation is a fraction of one percent when this method is used to describe the slopes of a wind ruffled surface. An extension of the method to image both components of slope requires two arrays of lamps pulsed alternately and in synchronism with a fast camera.

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References and links

Introduction

The shape of the sea surface on all scales is of importance as the roughness regulates the structure and intensity of turbulence in the wind above the sea. These factors in turn control the stresses that the wind exerts on the ocean and the growth of wind waves. The shape also controls the transmission and reflection of sunlight and the scattering of microwaves.

Waves of the sea surface have a wide range of scales ranging upwards from a millimeter or less. Waves of large scale, swell and wind waves, have been effectively delineated by many methods including arrays of pressure sensors below the sea surface and scanning laser altimeters from above. Small scale waves are difficult to measure because measuring apparatus tends to distort the waves directly or to modify the air flow which generates them. The difficulties are illustrated by equipment used in the open sea to study the shortest waves [1–3]. All three measure the refraction of light rays from bright light sources by the slopes of the water surface in order to avoid disturbing the surface. Although they successfully resolve the finest scales of the sea surface, they require large support devices close to the water surface that must necessarily disturb water and air flow, and also generate unwanted reflected waves in the region of interest. Another approach that avoids contact with the water involves polarimetric imaging of light reflected from the sea surface [4]. This method uses as a source skylight, preferably from a cloudy sky. The principal limitation is the low intensity of reflected skylight. The movement of short waves during needed exposure times smears the images of the shortest waves. We describe here a method in which bright optical sources and receivers can be kept at considerable distances from the water air interface to avoid disturbances.
Fig. 1. (a) A compressed image of 10 colored fluorescent lamps viewed through the still water surface in a laboratory wind/wave channel. Each lamp is 16 mm in diameter and 1.15 m long. They are separated by 74 mm center to center and are effectively 3 m below the water surface. (b) Part of an image of the lamps when a wind of 3.3 m/s (channel averaged) blows over the water surface. The x axis is parallel to the long axis of the image and the lamps are parallel to the y axis. The area of water surface shown is 176 mm by 80 mm (2816 by 1287 pixels). The camera, mounted with entrance pupil 1.8 m above the mean water surface, was pointed straight down. The photograph is one of 60 exposed in a burst one second long with exposure times of one millisecond. The image has been enhanced by a process that clears the background and identifies the color of individual contour stripes by averaging the color coordinates within each stripe. Stripes containing fewer than 128 pixels are ignored.

Contouring one component of slope

We mount a horizontal array of long, thin, colored lamps below the water surface, to form the appearance of contours as viewed from above. By mounting the lamps well below the water surface and the optical receiver well above we allow free water and air flow adjacent to the water surface. Each lamp is oriented normal to the direction corresponding to the component of slope to be sensed. For example, if x and y are horizontal components, lamps parallel to the y axis produce contours of the x component of slope. Viewed from above, when the water surface is flat, the lamps appear as almost straight colored lines (Fig. 1(a)). When the water surface is ruffled by wind the lamp images are broken into multiple curved and colored stripes (Fig. 1(b)). In this example, x is positive downwind and the lamps are parallel to the y direction. Each color provides, with a slight degree of uncertainty, a contour of x-slope, the value of which varies slowly in the x direction. We shall refer to such stripes as ‘contour stripes’. The uncertainty arises because the true value of x-slope at each point within a contour stripe depends to some extent on the y component of slope at which the refraction of the light ray occurs.
Uncertainty of contour value within a contour stripe

The contour represented by each contour stripe and its uncertainty can be calculated in the following way. Let \( \mathbf{N} \) be an upward directed unit vector normal to the water surface at \( p(x,y,z) \) (Fig. 2). Here the rectangular coordinate system \((x,y,z)\) is centered on the entrance pupil of the camera, with \(z\)-axis upwards. \( \mathbf{N} \) has components \((\cos \alpha \sin \theta, \sin \alpha \sin \theta, \cos \theta)\), where \( \theta \) is the vertical angle of \( \mathbf{N} \) and \( \alpha \) its azimuth. Snell’s law for refraction of light rays is contained in the expression

\[
n \mathbf{I} - \mathbf{R} = k \mathbf{N},
\]

where \( n \) is the index of refraction of the water relative to air, \( \mathbf{I} \) and \( \mathbf{R} \) are unit vectors parallel to the incident and refracted rays respectively, and \( k \) is positive. Total internal reflection of the incident ray occurs when \( \mathbf{I} \cdot \mathbf{R} \leq 1/n \). In terms of the angles summarized in Fig. 2,

\[
\mathbf{I} = (\cos \beta \sin \phi, \sin \beta \sin \phi, \cos \phi),
\]

\[
\mathbf{R} = (\cos \gamma \sin \psi, \sin \gamma \sin \psi, \cos \psi).
\]

The three components of Eq. (1) are contained in

\[
n \cos \beta \sin \phi - \cos \gamma \sin \psi = k \cos \alpha \sin \theta, \quad (2a)
\]

\[
n \sin \beta \sin \phi - \sin \gamma \sin \psi = k \sin \alpha \sin \theta, \quad (2b)
\]

\[
n \cos \phi - \cos \psi = k \cos \theta. \quad (2c)
\]
Since \( k \sin \theta \) is positive, Eqs. (2a) and (2b) have the signs of \( \cos \alpha \) and \( \sin \alpha \) respectively, and \( \alpha \) can be found from the four quadrant arc tangent of the ratio of Eq. (2b) to Eq. (2a). The sum of squares of Eq. (2a) and Eq. (2b) divided by the square of Eq. (2c) leads to the square of the tangent of \( \theta \). The \((x, y)\) components of slope of the water surface at \( p \) are \(-\cos \alpha \tan \theta \) and \(-\sin \alpha \tan \theta \) respectively.

The vertical component of distance from the camera entrance pupil to the image point on the water surface, \( h \), must be measured by some means (for example, acoustically) in order to relate pixel coordinates of image points, thus \((\psi, \gamma)\), to the corresponding \((x, y)\) points on the water surface. In addition the vertical component of distance \( H \) to the light source must be known. From these four input data together with the function \( S(X, Y, -H) \) describing the shape of the light source we can compute slope components of the water surface which could possibly contribute to every point within a contour stripe at water surface position, \((x, y)\). In general we find that there is a range of true contour values within each contour stripe. The resulting uncertainties of true values are illustrated in Figs. 3 and 4.

![Fig. 3. The relation of true contours to contour stripes. produced by a single lamp parallel to the y axis and centered below the camera (X = 0). The field of view of the camera is assumed to be 0.30 m by 0.24 m in the x and y dimensions respectively. (a) The mean value of each x-slope contour stripe produced by this lamp depends essentially only on x. (b) The uncertainties of contours expressed as the standard deviation of true values within each contour stripe found by the first method discussed in the text. (c) Standard deviation of probable uncertainties by the second method.](image)

Here it is assumed that the camera is pointed vertically downward, that its entrance pupil is \( h = 2.5 \) m above the water surface (thus assuming the waves on the surface are negligibly high), and that the light source is linear, 1.5 m long, parallel to the y axis and 2.5 m below the water surface. In Fig. 3 the center of the lamp is vertically below the camera \((X = 0)\). In Fig. 4 it is displaced 1.0 m \((X = 1.0)\). With either configuration the contour stripes are very nearly true contours of \( x \)-slope which vary slowly in the \( x \) direction but with a degree of ambiguity depending on the range of possible values of \( y \)-slope. There would be no ambiguity if the \( y \)-slopes were known within the contour stripe. Without such knowledge we show the uncertainty in two ways. The first assumes that rays of light entering the camera may have originated anywhere along the entire length of the lamp. The uncertainties are represented by...
the standard deviation of \( x \)-slope for an even distribution of light rays originating along the lamp such that \(|y| < 0.75 \) m. This would imply a very wide and unlikely range of \( y \)-slopes. The other method weighs the deviations in \( x \)-slope according to the probability of those \( y \)-slopes required to produce rays from any position along the lamp. The \( y \)-slope probability is taken as the nearly Gaussian form found in the open sea [5]. The variances of up/downwind and crosswind slope components increase linearly with wind speed. The former is larger than the latter and reaches a value of 0.05 at a wind speed of 15 m/s. As a consequence the ambiguity in \( x \)-slope contour stripes is a maximum when the wind is parallel to the \( y \) direction and increases with wind speed. The slope probability is known to be skewed: highest slopes are directed upwind and the most probable slope is a small value directed downwind. As a result, when the wind is coming from the positive \( y \) direction the uncertainties in \( x \)-slope contour stripes are larger at positive \( y \) locations than at negative \( y \) locations. This is illustrated in Figs. 3(c) and 4(c) where it is assumed the wind is blowing from the positive \( y \) direction at 15 m/s (as measured 10 m above the sea). The uncertainties would be reduced at lower windspeeds and when the wind direction is along the \( x \) axis.

\[ \text{Fig. 4. Similar to Fig. 3 except that the single lamp source is now at } X = 1.0 \text{ m.} \]

The uncertainties shown are roughly one percent of the mean contour value by the first method and one half of one percent from the second method.

**Lamp spacing and reconstruction from contours**

In order to reconstruct the entire image field of \( x \)-slopes it is necessary to interpolate between contours. Experience, to be related elsewhere, shows that errors inherent in interpolation are related to the ratio of contour spacing to rms slopes and to the extension of the slope spectrum into short wavelengths. The distribution of errors is sharply peaked at zero near contour stripes and reaches maxima far from the contours. In windblown water waves in a laboratory channel where the slope spectrum is sharply limited by viscosity, 95% of errors are less than 8% of rms \( x \)-slope when the contour interval is 80% of rms slope. As a result of this favorable circumstance, ten colored lamps are sufficient to create contour stripes covering a useful field of view and a wide range of slopes.
Exposure time and evolution of slopes in time

Slope contours of wind waves move at speeds comprised of the inherent phase speeds of capillary waves superposed on the orbital velocities of underlying gravity waves. In laboratory wind wave channels this speed reaches 0.5 m/s. For this reason exposure times must be as short as one millisecond. The focal ratio needed for adequate depth of focus (f/8) then puts a lower limit to lamp intensity. Fifty watt fluorescent lamps have been found to provide sufficient light intensity to meet the need.

Contouring both slope components

The contour stripes shown in Fig. 1(b) were generated by an array of fluorescent lamps that are inherently steady state devices. The recent development of strips of high power light emitting diodes that can be switched on and off at millisecond rates presents the opportunity of switching between two arrays of lamps, one y parallel and one x parallel. A single high speed camera, with exposures synchronized with the lamps can then image x-slope and y-slope contour stripes in alternate images.

Spectral range

Our method requires resolution of individual contour stripes. The separation of contours is governed by very short waves. On a windblown water surface capillary waves with wavenumbers extending up to 2 per mm (wavelength 3 mm) have been resolved when the image magnification in the camera corresponds to 16 pixels per millimeter on the water surface, and contour stripes are well separated. The long wavelength limit is then set by the number of pixels in each dimension of the full image. Figure 1(b) was made by a camera with 2816 by 2112 pixels in the x and y directions. Thus the longest waves to be encompassed in the image were 176 mm and 132 mm in the x and y directions (wavenumbers 0.036 and 0.048 per mm). Multiple cameras or cameras with higher resolution can extend these long wavelength limits.

Possible use at sea

The contouring method described here has the advantage for open sea work that both light sources and camera or cameras can be kept far enough from the sea surface to avoid disturbing air flow, water flow or waves in the region of measurement. The requirements for a platform able to support the sources and cameras at sea are stringent. It should also not affect wind, waves or currents in the region of interest and must follow the vertical motions of large wind waves and swells to maintain the geometry of sources and cameras with some degree of uniformity.

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