



## Adriatic seiche decay and energy loss to the Mediterranean

IVANA CEROVEČKI\*, MIRKO ORLIĆ† and MYRL C. HENDERSHOTT‡

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**Abstract**—A salient feature of sea level records from the Adriatic Sea is the frequent occurrence of energetic seiches of period about 21 h. Once excited by a sudden wind event, such seiches often persist for days. They lose energy either to friction within the Adriatic, or by radiation through Otranto Strait into the Mediterranean.

The free decay time of the dominant (lowest mode) seiche was determined from envelopes of band-passed sea level residuals from three locations (Bakar, Split and Dubrovnik) along the Croatian coast during twelve seiche episodes between 1963 and 1986 by taking into consideration only time intervals when the envelopes decreased exponentially in time, when the modelled effects of along-basin winds were smaller than the error of estimation of decay time from the envelopes and when across-basin winds were small. The free decay time thus obtained was  $3.2 \pm 0.5$  d. This value is consonant with the observed width of the spectral peak.

The decay caused by both bottom friction and radiation was included in a one dimensional variable cross section shallow water model of the Adriatic. Bottom friction is parameterized by the coefficient  $k$  appearing in the linearized bottom stress term  $k\rho_0u$  (where  $u$  is the along-basin velocity and  $\rho_0$  the fluid density). The coefficient  $k$  is constrained by values obtained from linearization of the quadratic bottom stress law using estimates of near bottom currents associated with the seiche, with wind driven currents, with tides and with wind waves. Radiation is parameterized by the coefficient  $a$  appearing in the open strait boundary condition  $\zeta = auh/c$  (where  $\zeta$  is sea level,  $h$  is depth and  $c$  is phase speed). This parameterization of radiation provides results comparable to allowing the Adriatic to radiate into an unbounded half plane ocean. Repeated runs of the model delineate the dependence of model free seiche decay time on  $k$  and  $a$ , and these plus the estimates of  $k$  allow estimation of  $a$ .

The principle conclusions of this work are as follows.

(1) Exponential decay of seiche amplitude with time does not necessarily guarantee that the observed decay is free of wind influence.

(2) Winds blowing across the Adriatic may be of comparable importance to winds blowing along the Adriatic in influencing apparent decay of seiches; across-basin winds are probably coupled to the longitudinal seiche on account of the strong along-basin variability of across-basin winds forced by Croatian coastal orography.

(3) The free decay time of the 21.2 h Adriatic seiche is  $3.2 \pm 0.5$  d.

(4) A one dimensional shallow water model of the seiche damped by bottom stress represented by Godin's (1988) approximation to the quadratic bottom friction law  $\rho_0 C_D u|u|$  using the commonly accepted drag coefficient  $C_D = 0.0015$  and quantitative estimates of bottom currents associated with wind driven currents, tides and wind waves, as well as with the seiche itself with no radiation gives a damping time of 9.46 d; radiation sufficient to give the observed damping time must then account for 66% of the energy loss per period. But independent estimates of bottom friction for Adriatic wind

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\* Rudjer Bošković Institute, Center for Marine Research, Bijenička 54, 41000 Zagreb, Croatia.

† Andrija Mohorovičić Geophysical Institute, Faculty of Science, University of Zagreb, Horvatovac bb, 41000 Zagreb, Croatia.

‡ Scripps Institution of Oceanography 0209, University of California in San Diego, 9500 Gilman Drive, La Jolla CA 92093-209, USA.

driven currents and inertial oscillations, as well as comparisons between quadratic law bottom stress and directly measured bottom stress, all suggest that the quadratic law with  $C_D = 0.0015$  substantially underestimates the bottom stress. Based on these studies, a more appropriate value of the drag coefficient is at least  $C_D = 0.003$ . In this case, bottom friction with no radiation leads to a damping time of 4.73 d; radiation sufficient to give the observed damping time then accounts for 32% of the energy loss per period. © 1998 Elsevier Science Ltd. All rights reserved

## INTRODUCTION

The Adriatic Sea is an elongated (about 800 km by 200 km) semi-enclosed basin which communicates with the Mediterranean through the narrow (75 km) and shallow (325 m mean depth) Otranto Strait. The Adriatic is deep enough that the contribution of bottom friction to the horizontal momentum balance is small and Otranto is narrow enough that radiation is small; this situation supports the existence of persistent free oscillations. Through most of the year (with the exception of the summer) the Adriatic Sea is under the atmospheric westerlies, resulting in frequent passages of cyclones generated above the Mediterranean Sea. Consequently, surges followed by energetic oscillations of the lowest basin mode seiches are prominent features of mareographic records.

If the surge coincides with the time of high tide, sea level in the northern Adriatic can reach very high values; the phenomenon being known as "acqua alta". The most severe one, in November 1966, caused widespread damage in Venice, where sea level reached values up to 147 cm (Finizio *et al.*, 1972). In order to be able to forecast such events, a number of storm-surge models have been developed: one dimensional shallow water models both frictionless (Robinson *et al.*, 1973) and with linear friction (Accerboni *et al.*, 1971; Finizio *et al.*, 1972; Tomasin, 1973) and two dimensional models with either linear bottom friction (Stravisi, 1972, 1973) or quadratic bottom friction (Accerboni and Manca, 1973; Michelato, 1975).

Adriatic seiches have been extensively investigated from the beginning of this century: von Kesslitz (1910) noticed periodic motions with 23 h and 12 h period in detided sea level records from Pula. Defant (1911) computed periods of the Adriatic free oscillations and reported values of 22.4 h and 12 h. After that many authors estimated periods of several of the lowest longitudinal modes from residual sea level using different methods and obtained considerably different results (e.g. Defant, 1960; Buljan and Zore-Armanda, 1976). Widespread use of spectral techniques after about 1970 resulted in estimates more similar to one another; these will be discussed below.

At the same time, periods were also estimated by analytical and numerical models. Bajc (1972) computed periods of 21.0 h, 12.5 h and 8.7 h for the three lowest mode seiches using a channel model. Sguazzero *et al.* (1972) calculated corresponding periods using a one dimensional analytical model and reported values of 21.7 h, 9.9 h and 6.5 h. Accerboni and Manca (1973) and Michelato (1975) computed periods of 20.6 h and 20.8 h for the fundamental longitudinal seiche using a two dimensional model. Poretti (1974) calculated the period of the fundamental seiche with a model in which the Adriatic was considered as a rectangular basin. His estimated value of 19.2 h did not change after bottom friction was introduced, but after rotation was introduced it decreased to 18.6 h. Michelato *et al.* (1985) investigated periods and spatial structure of seiches using a two dimensional model and forcing the basin through the Otranto Strait with a set of waves with periods from 5 min to 30 h.

Adriatic models that do not encompass the entire Mediterranean must assume an open boundary condition. The importance of the choice made is stressed by Schwab and Rao

(1983). They modeled the whole Mediterranean, taking into account rotation and real topography and estimated the position of nodal lines for the three lowest longitudinal Adriatic modes. Thereafter they calculated the corresponding periods by retaining those nodal lines in a model of the Adriatic only, with a finer grid and without rotation; they reported values of 21.9 h, 10.7 h and 6.7 h.

Although Adriatic seiches have been extensively investigated, only one paper has been found analyzing the damping of the seiches in detail. Godin and Trotti (1975) modelled six major seiches, identified in detided sea level records from Trieste between 1952 and 1971, as damped sinusoids from which the approximate decay time of such events was estimated.

The purpose of this study is to estimate free decay times and to attempt to distinguish between the dissipation of energy within the Adriatic due to bottom friction and the radiation of energy through the Otranto Strait into the Mediterranean. With this aim in view, the actual decay times of first mode seiches were carefully determined and screened for wind contamination. Using a one dimensional variable cross section shallow water model, the dependence of model seiche decay time on bottom friction and radiation was delineated. Finally, using the empirically obtained free decay time, the model results and *a priori* estimates of bottom stress using estimates of near-bottom wind driven currents, tides and wind wave orbital velocities as well as currents associated with the seiche, the fraction of energy transmission through Otranto was estimated.

## DATA DESCRIPTION AND ANALYSIS

### *Data*

In order to analyse the decay of Adriatic seiches, hourly sea level data, recorded at Bakar, Split and Dubrovnik (Fig. 1) between 1963 and 1986 were gathered. Since it is generally assumed that the atmospherically forced motions are linearly superimposed on tides in the Adriatic Sea, residuals were calculated by subtracting the predicted tide from observed sea level. Seven tidal constituents were taken into account (e.g. Kasumović, 1952). Tidal harmonic constants were read from Tide tables (Hydrographic Institute, 1973), astronomical constants from Schureman (1941) and mean sea level data from publications of the Hydrographic Institute (1964–1987). Twelve episodes of seiches, characterized by an initial amplitude that exceeded 45 cm at Bakar, were chosen for further analysis. Table 1 lists these episodes with the corresponding dates. Figure 2 shows residuals during episode 5 in November 1966. Residual levels recorded during this episode were the highest in the observations analysed; after the cold front swept across the Adriatic, energetic seiches lasting about thirteen days were generated. We chose this episode as an example to be shown after each step in the data analysis procedure.

### *Sea level spectra*

Spectra of both observed and residual sea levels were calculated for each episode to check the accuracy of extracting the tidal signal. The trend was removed from each time series and a cosine window applied. Energy spectra were computed via the FFT method with 4 degrees of freedom and a frequency resolution of 0.0017 cph. Figure 3 shows them for episode 5; the diurnal tide was successfully removed and the energy at the semidiurnal frequency was considerably reduced (leaving the nearby peaks, due to seiches as will be shown next, unaffected).



Table 1. Episodes during which prominent seiches were observed from 1963 to 1986

Episode	Interval over which seiches were observed	Interval selected for spectral analysis
1	01 Feb–18 Feb, 1963	04 Feb–18 Feb, 1963
2	09 Oct–24 Oct, 1964	09 Oct–23 Oct, 1964
3	30 Nov–16 Dec, 1964	02 Dec–16 Dec, 1964
4	18 Jan–06 Feb, 1966	23 Jan–06 Feb, 1966
5	30 Oct–22 Nov, 1966	05 Nov–22 Nov, 1966
6	12 Feb–06 Mar, 1967	15 Feb–01 Mar, 1967
7	02 Jan–19 Jan, 1968	05 Jan–19 Jan, 1968
8	11 Feb–01 Mar, 1969	15 Feb–01 Mar, 1969
9	28 Feb–14 Mar, 1974	28 Feb–14 Mar, 1974
10	16 Nov–01 Dec, 1977	17 Nov–01 Dec, 1977
11	09 Dec–24 Dec, 1981	10 Dec–24 Dec, 1981
12	26 Jan–10 Feb, 1986	27 Jan–10 Feb, 1986

Let us briefly compare the estimated first and second mode periods with some values in the literature obtained by spectral analysis of residual sea levels. Sguazzero *et al.* (1972) reported periods of 21.7 h and 10.8 h of the first and the second Adriatic mode respectively, Manca *et al.* (1974) 21.3 h and 10.8 h, Poretti (1974) 22.0 h and 10.8 h, Godin and Trotti

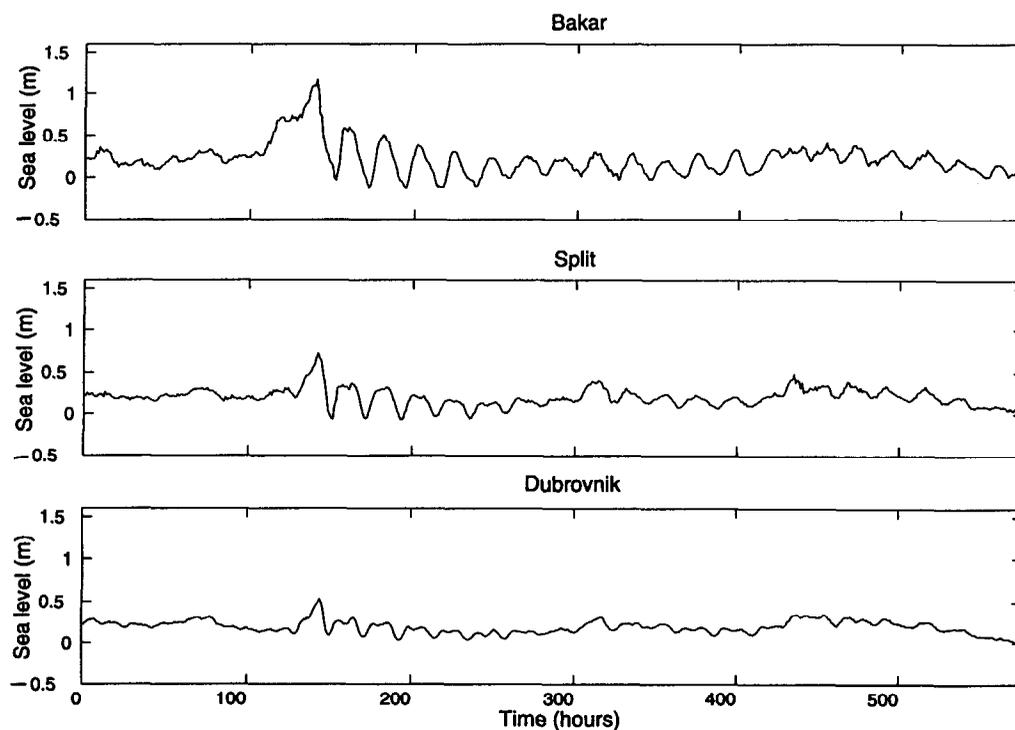


Fig. 2. Residual sea levels (m) at Bakar, Split and Dubrovnik for episode 5 (Table 1) between 30 October and 22 November 1966.

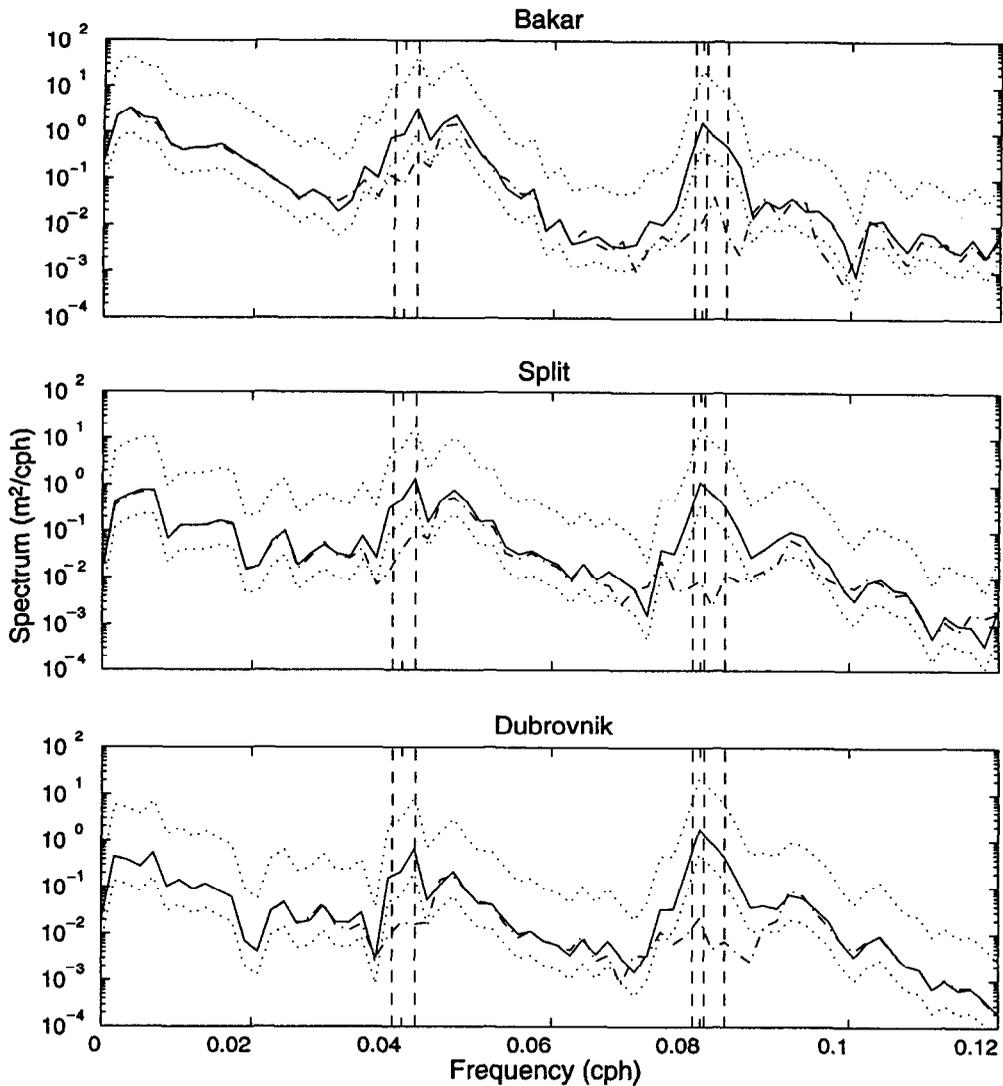


Fig. 3. Spectra ( $\text{m}^2/\text{cph}$ ) of tide gauge records (solid line; dotted lines enclose 99% confidence interval) and of residual sea levels (dot-dashed line) at Bakar, Split and Dubrovnik for episode 5 (Table 1) between 30 October and 22 November 1966. Vertical dashed lines indicate (left to right) frequencies of O1, K1, N2, M2, S2 tidal lines.

(1975) 21.5 h and 11.1–11.4 h and Mosetti and Purga (1983) 21.4 h and 10.8 h. Obviously, agreement with the present values of 21.2 h and 10.9 h is good.

The least pronounced peak is at the 12.4 h period. Its amplitude varied appreciably from episode to episode, but usually the amplitude is highest at Bakar, smaller at Dubrovnik, smallest at Split. This peak is very close in frequency to the M2 tide, the largest in the Adriatic, suggesting some connection between the two. Several researchers have reported an oscillation at 12.5 h period (Sguazzero *et al.*, 1972; Manca *et al.*, 1974; Mosetti and Purga, 1983) attributing it to tidal energy that had passed through the filters.

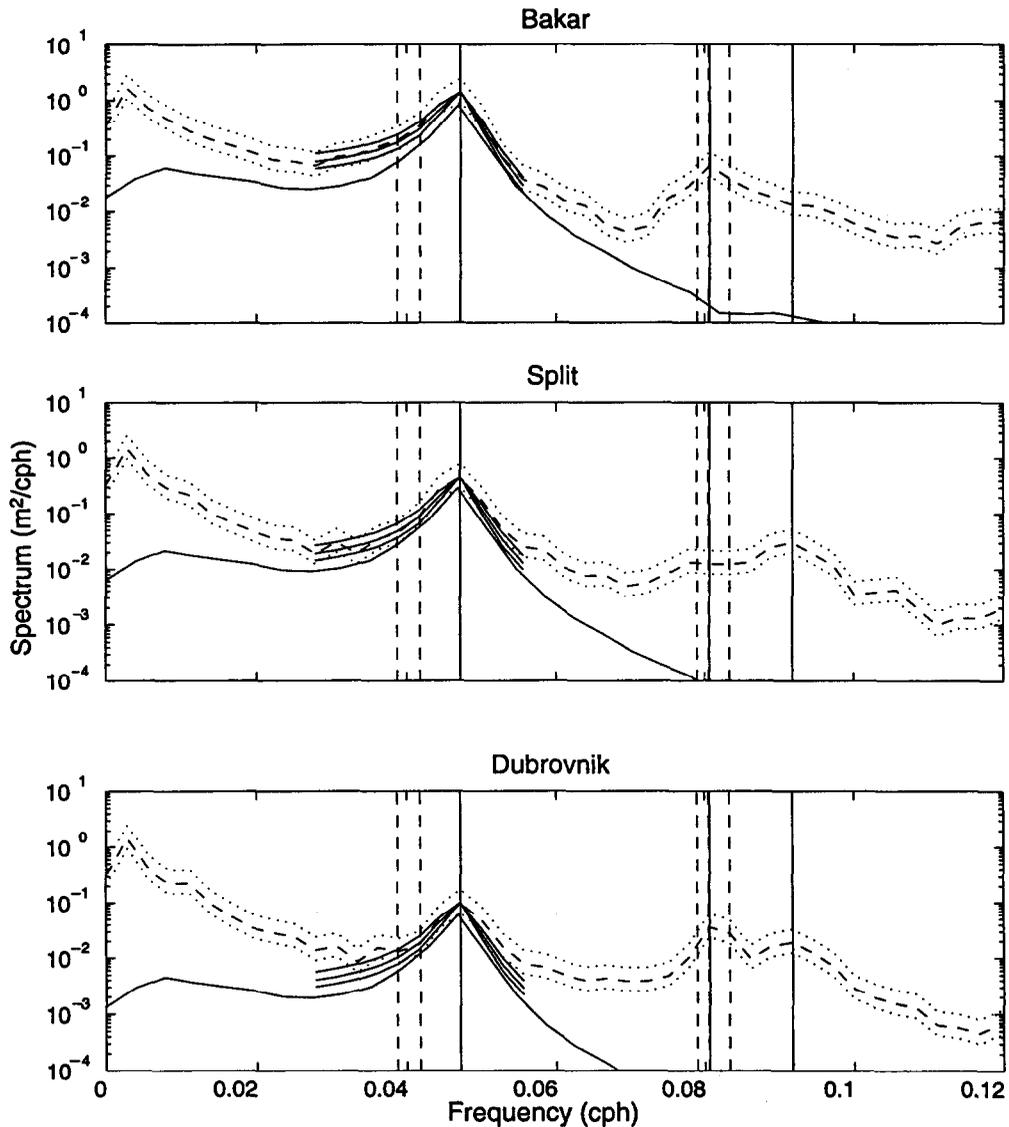


Fig. 4. Dashed line shows mean spectra ( $\text{m}^2/\text{cph}$ ) of 15-day long subsets of residual sea levels with seiches at Bakar, Split and Dubrovnik during the 12 episodes analysed. Dates are given in Table 1. Dotted lines enclose 99% confidence interval. Solid lines show spectrum ( $\text{m}^2/\text{cph}$ ) of damped harmonic oscillator (damping times  $3.2 \pm 0.5$  d) driven by random forcing as described in text. Periods corresponding to three significant peaks indicated by vertical solid lines are 21.2 h, 12.4 h, and 10.9 h. Vertical dashed lines indicate (left to right) frequencies of O1, K1, N2, M2, S2 tidal lines.

We wondered if this semidiurnal peak (near 0.0806 cph) might be the result of nonlinear interaction between diurnal tides and the 21.2 h seiche. Fourier analysis of 32768 hours of detided sea level records at Bakar, Split and Dubrovnik showed small but significant spectral peaks at the frequencies corresponding to quadratic nonlinear interactions between the most important diurnal constituents K1, P1 and O1. Similar nonlinear interactions

between the diurnal seiche and the diurnal tides would result in spectral peaks centered around the frequencies 0.0859 cph, 0.0887 cph, 0.0890 cph (corresponding to the sum of the seiche central frequency 0.0472 cph and one of the diurnal tidal frequencies O1, P1, K1) and having the same width as the seiche peak. If the energy in the seiche were always the same as the energy of a diurnal line, then the energy under the seiche–tide peak would be the same as the energy in the corresponding tide–tide line. But the energy in the seiche is usually a good deal smaller than the energy in the diurnal tide and in any event the diurnal constituent interaction lines are not large; correspondingly we should not have expected to find significant spectral peaks due to nonlinear interaction of the seiche with the diurnal tides and we did not see them.

### *Estimation of the free decay time of the 21.2 h seiche*

In all the spectra calculated, the first mode (estimated frequency 0.0472 cph) was characterized by the highest energies. In order to extract this mode from the hourly residual sea level series, a band-pass filter was used. The cutoff frequencies were 0.0344 cph and 0.0594 cph and the half-length of the filter equalled 80 hr. The filter weights were computed according to Cartwright (1970).

The final aim of the empirical analysis was accurate estimation of the free decay time of the lowest Adriatic mode, the decay time to which dynamical significance can be assigned. With this aim in view envelopes of band-pass filtered residuals were calculated (Farnbach, 1975). The bottom friction-induced decay of nearly periodic motions, such as seiches, is expected to be exponential (third section). The envelopes were therefore plotted on a logarithmic scale. Figure 5 gives an example of how departure of the logarithm of the envelope from a straight line indicates when a new input of energy from the atmosphere begins. From this analysis, five intervals (Table 2) of apparent free decay during which the logarithm of the envelope decreased nearly linearly in time were selected from 12 episodes of Table 1.

For these five intervals the decay time, i.e. the time during which the amplitude of oscillations decreases by  $e^{-1}$  was computed. A function having the form  $A_0 \exp(-\alpha t)$  (where  $A_0$  is the initial amplitude, and  $\alpha^{-1}$  is the decay time) was least-squares fitted to the envelopes for the intervals of apparent free decay of Table 2 at Bakar, Split and Dubrovnik. Decay times thus obtained are given in Table 2. They differ sufficiently from one another that further analysis is required.

The interval selection procedure outlined above discriminates against forcing by strong wind impulses but cannot distinguish effects of normally present weak winds from free decay. In order to estimate such wind influence on the decay times obtained, sea level residuals during the episodes containing the selected intervals were reproduced with a linear one dimensional variable cross section shallow water model. Hourly values of wind data from Pula (Fig. 1), Split, and Dubrovnik were gathered and it was assumed that winds in the northern third of the Adriatic could be represented by winds at Pula, winds in the middle part of the basin by winds at Split and winds in the southern third of the basin by winds at Dubrovnik. Forward-time and central-space finite difference approximations were used to obtain difference equations, wind forcing was represented by the wind stress  $\rho_{oa} C_{DA} |W|W|$  ( $C_{DA}$  is the non-dimensional drag coefficient appropriate to the air–sea interface,  $\rho_{oa}$  the density of the air and  $W$  the along-basin wind component) and bottom friction was represented by the linear stress term  $k\rho_o u$  ( $k$  is the coefficient of bottom friction,  $\rho_o$  is the

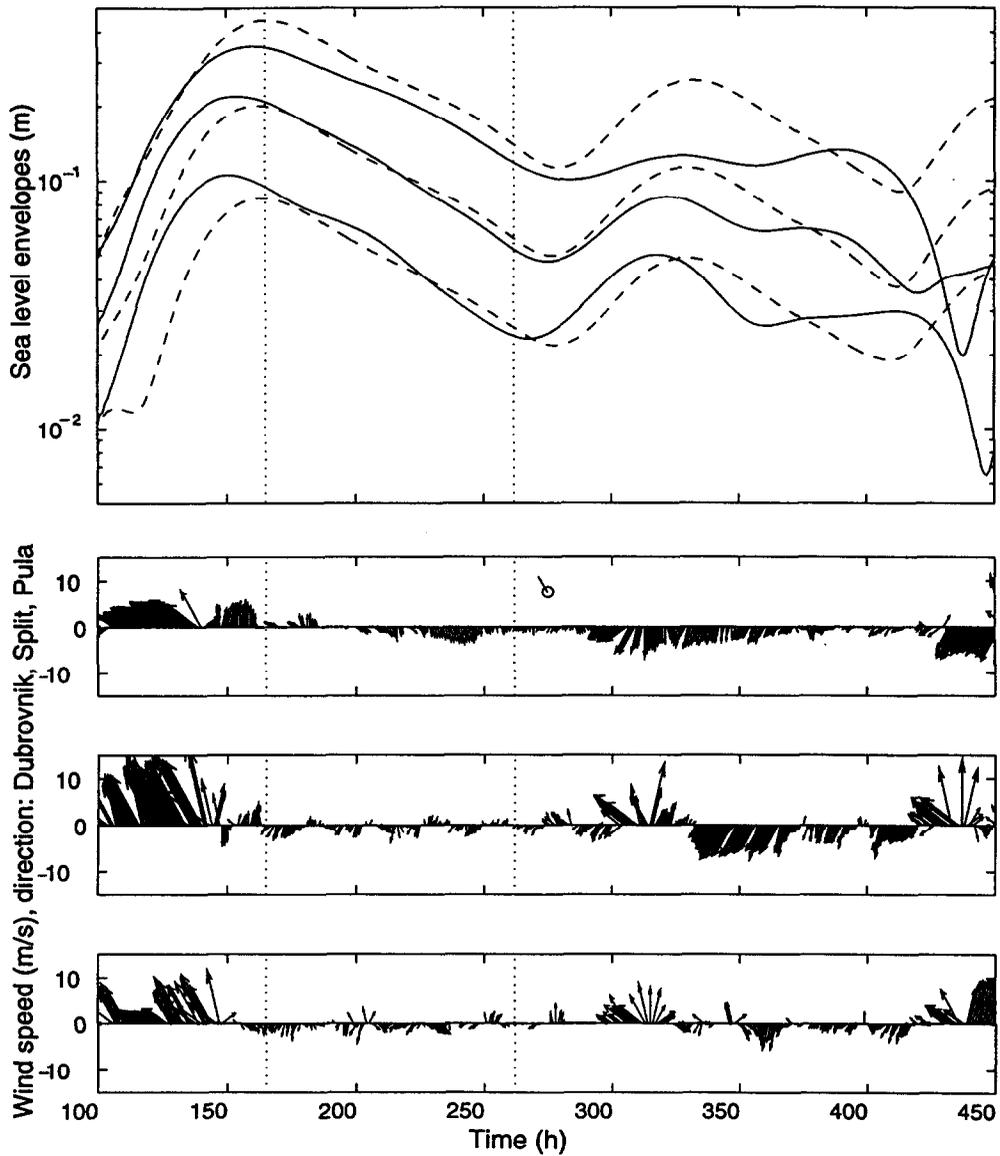


Fig. 5. Top panel: Envelopes (solid lines) of band-passed residual sea levels recorded at (top to bottom) Bakar, Split and Dubrovnik; envelopes (dashed lines) of corresponding model elevations. Bottom three panels: winds recorded at (top to bottom) Pula, Split and Dubrovnik, all for episode 5 (Table 1) between 30 October and 22 November 1966. Barb in Pula panel at hour 275 indicates along-basin direction. In all panels, dotted vertical lines bound intervals with nearly free oscillations. Departure of envelopes from linearity indicates a new input of energy from the atmosphere.

Table 2. Empirical values of free decay time for selected seiches

Episode	Start and end times of apparent free decay (hours after beginning of record)		Empirical decay times (d)		
			Bakar	Split	Dubrovnik
1a	117	148	3.22	2.58	1.23
1b	287	334	4.88	3.50	2.56
3	203	243	4.02	2.92	2.26
4	153	206	3.34	4.46	4.30
5	165	262	3.80	2.90	2.81
12	179	210	2.80	3.80	5.52

fluid density and  $u$  the along-basin velocity). Boundary conditions were no flow at the closed end and zero elevation at the open end. The calculation began from rest. The Adriatic was discretized according to von Sterneck (1919), with a space step of 20.5 km. We could not expect that the one dimensional model would *ab initio* predict the period of the seiche exactly on account of finite along channel resolution, neglected cross channel variation, rotation, etc. Nonetheless since observed winds were to be used to drive the model, it was essential that the model seiche period should be the correct one. Consequently, von Sterneck's depths were reduced by 6% in order to slightly change the seiche period and correspondingly improve the model predictions. The choice of the value  $k = 0.6 \times 10^{-3} \text{ ms}^{-1}$  used in the calculations of this section is further discussed below.

Model results were compared to observed residuals for the five selected episodes. This was done because in these later cases, the three sets of wind data used do describe well the wind field above the Adriatic Sea and the forcing is dominantly longitudinal. The influence of the along-basin wind component can thus be estimated from the difference between the model seiche decay times obtained with real winds and with winds set equal to zero after the seiche-generating wind impulse.

The only episode of the five during which modeled and observed sea level differed significantly was episode 3, when the model did not reproduce the initial surge well, nor consequently the following seiches. For the other four episodes hourly values of model results corresponding to the intervals of observed residuals selected above were analysed in the same way as empirical data. Both model seiche decay time with the real wind and with the wind set equal to zero after the seiche generating wind impulse were calculated and the values are given in Table 3.

Decay times obtained from "model without wind" results are virtually the same at Bakar, Split and Dubrovnik for intervals 1b, 4, 5 and 12, just as we expect if the band-passed model output is the fundamental mode only, but those decay times vary from interval to interval. Why? The answer cannot be "contributions from other modes" because the decay times have been calculated from band-passed model output. Rather the difference of decay times between different intervals indicates the uncertainty of estimates of decay times made by finding a period of time over which decay appears to be exponential and estimating the decay times from the logarithm of the envelope. The most similar decay time estimates are from intervals 1b, 4 and 5. In these, the exponential decay lasts several cycles whereas the most variable decay times are from the intervals 1a and 12 where it lasts little more than a cycle. When the analysing intervals are artificially extended to ten seiche periods by continuing to run the model without wind, all estimated decay times agree to a few percent.

Table 3

Episode 1a			
Bakar	3.22	2.05	3.95
Split	2.58	1.79	3.66
Dubrovnik	1.23	1.53	2.91
Episode 1b			
Bakar	4.88	2.57	3.05
Split	3.50	2.44	2.97
Dubrovnik	2.56	2.26	2.96
Episode 4			
Bakar	3.34	3.13	3.05
Split	4.46	3.11	2.96
Dubrovnik	4.30	3.03	2.96
Episode 5			
Bakar	3.80	3.39	3.17
Split	2.90	3.26	3.16
Dubrovnik	2.81	3.29	3.17
Episode 12			
Bakar	2.80	4.89	2.75
Split	3.80	4.69	2.67
Dubrovnik	5.52	4.99	2.75

First column: free decay times (d) determined empirically from intervals of apparent free decay in sea level residuals, as for Table 2. Second column: decay times (d) determined from model sea level driven by actual winds blowing from beginning of episode to end of interval of apparent free decay in sea level residuals. Third column: decay times (d) determined from model sea level driven by actual winds from beginning of episode to beginning of apparent free decay in sea level residuals.

We thus conclude that our method of estimating decay times may yield results that are in error by about 0.5 d for the rather short periods of free decay occurring in the observations.

Comparison of “model with wind” and “model without wind” results indicates that intervals 4 and 5 show the smallest influence of longitudinal winds on decay time estimates (within the error of 0.5 d). Wind influence in episode 1b is only slightly greater and it is very large in the episodes 1a and 12. This is exactly what would be concluded from examining wind records for these intervals. One can see from the intervals 1a and 12, that even in the cases where the envelopes appear to decay exponentially, effects of longitudinal winds on decay time estimates can be as much as several days, much bigger than error ascribed to the method used.

Comparison of decay times obtained from “model with wind” and “model without wind” results shows that intervals 4 and 5 are the least influenced by longitudinal winds. Therefore we might expect the residual sea level decay times would be the same for these two intervals, but they are not: decay times at Split and Dubrovnik are a day and a half longer for interval 4 than for interval 5. Why? The seiche-generating wind event of interval 4 is followed by equally strong transverse winds that blew during the whole period of exponential decay (Fig. 6), whereas the generating wind event of interval 5 was followed by nearly complete

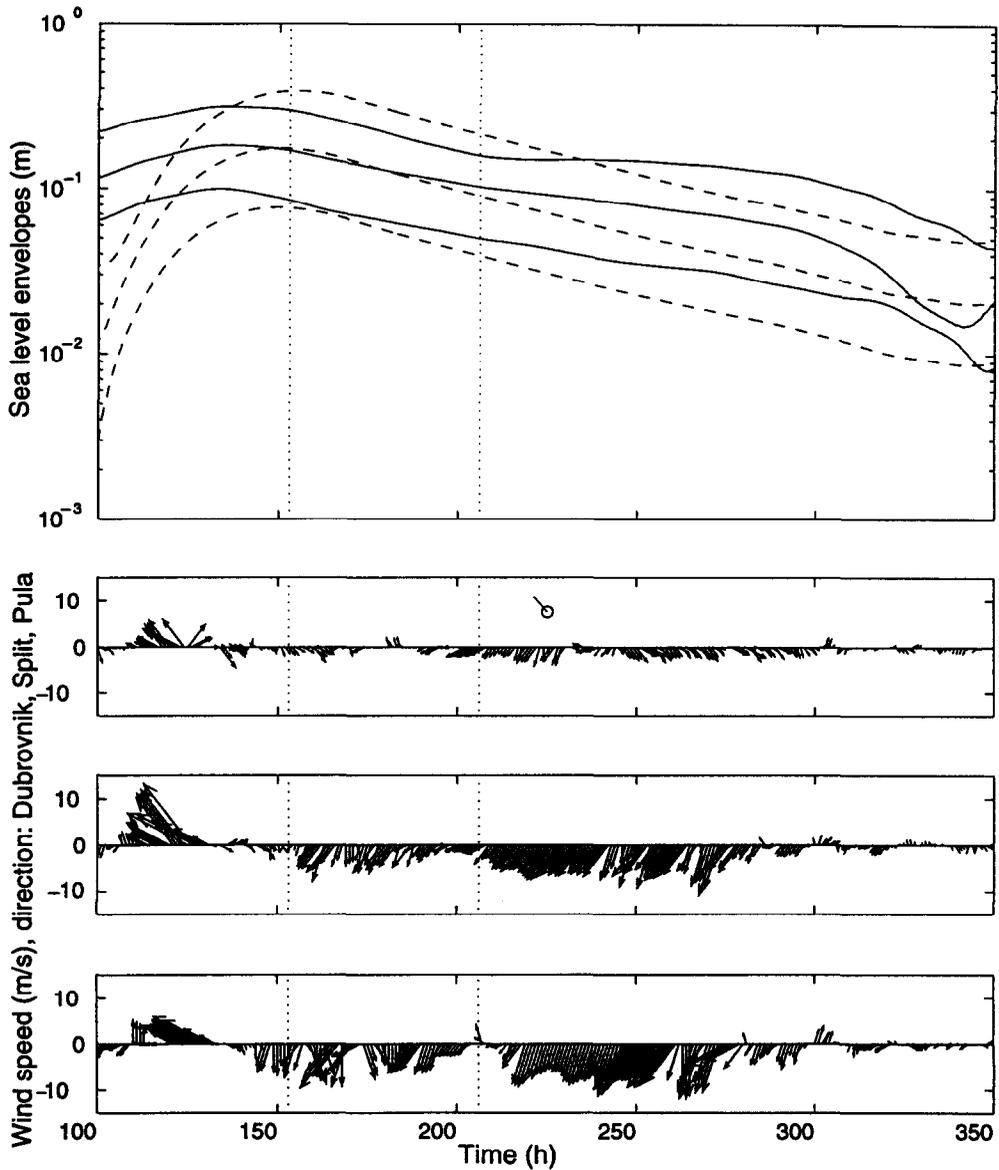


Fig. 6. Top panel: Envelopes (solid lines) of band-passed residual sea levels recorded at (top to bottom) Bakar, Split and Dubrovnik; envelopes (dashed lines) of corresponding model elevations. Bottom three panels: winds recorded at (top to bottom) Pula, Split and Dubrovnik, all for episode 4 (Table 1) between 18 January and 6 February 1966. Barb in Pula panel at hour 225 indicates along-basin direction. In all panels, dotted vertical lines bound intervals with nearly free oscillations. Comparison of winds with differences between envelopes of observed and modeled elevations suggests that across-basin winds significantly influence longitudinal seiches.

calm (Fig. 5). Consequently, one can conclude that the decay times of interval 4 are strongly influenced by transverse winds — not the only case in which they seem to be important.

We are thus left with interval 5 as the only case that is free of effects of longitudinal winds (by model test) and of effects of transverse winds (by inspection of wind records). For interval 5 the decay time estimates at Bakar, Split and Dubrovnik are (albeit barely) within the error of  $\pm 0.5$  d inherent in our envelope procedure. We thus conclude that the best estimate we can make of the free decay time is  $3.2 \pm 0.5$  d.

In order to obtain an independent check of this value, we followed the suggestion of Garrett and Munk (1971) that the decay time could also be estimated by fitting, to the observed sea level spectrum, the theoretical shape for the response of a damped oscillator driven by white noise. We made such a fit to the spectra of Fig. 4; the spectrum of the driving process at frequencies within the seiche peak (0.0278 cph to 0.0694 cph) was approximated not by a white noise spectrum but by a straight line between observed spectral values at these two frequencies multiplied by a constant chosen to give the observed response at the fundamental mode seiche frequency (0.0472 cph). The spectra thus obtained are plotted in Fig. 4 for our empirically determined decay times  $3.2 \pm 0.5$  d. The agreement between theoretical and observed spectra for the 3.2 d decay time is the best at Bakar and Split, better than at Dubrovnik, where the signal to noise ratio is smaller.

The foregoing shows that the width of the sea level spectral peak associated with the seiche is consonant with our estimated damping time of  $3.2 \pm 0.5$  d. It is of interest to further ask if the damped harmonic oscillator model correctly predicts the overall spectral level from the wind stress spectrum. If the depth of the basin is  $h(x)$ , if the wind stress is idealized to have no along basin variation and if the along basin velocity field associated with the seiche is idealized to have the spatial form  $u_o(x)$  of the lowest open-basin mode (quarter-wave), then the relationship between the spectrum  $S_Z(\sigma)$  of sea level and the spectrum  $S_T(\sigma)$  of wind stress is

$$S_Z(\sigma) = [S_T(\sigma)/\rho_o^2] \{ [d(u_o h)/dx] [ \int u_o dx ] / [ \int h u_o^2 dx ] \}^2 / [(\sigma^2 - \sigma_o^2)^2 + (\sigma R)^2]$$

in which  $\sigma_o$  is the seiche radian frequency and  $R = 2/(\text{damping time})$ .  $S_T(\sigma)$  was estimated from wind records associated with five of the most intense seiching episodes, and  $u_o(x)$  and its integrals were estimated from the one dimensional finite difference model. The resulting estimate of  $S_Z(\sigma)$  is plotted in Fig. 4 for a damping time of 3.2 d. It reproduces both the shape and the along-basin dependence of the observed spectra near the seiche frequency, although its amplitude is systematically low by a factor between one and two, not surprising in view of the assumed constancy of the wind stress over the basin.

Although many authors have investigated the Adriatic seiches, there were very few papers, to our knowledge, that addressed their decay. Sguazzero *et al.* (1972) filtered residual sea levels from several locations in the Adriatic Sea during 1966 using a band-pass filter centered around 21.7 h. They noticed that the amplitude of the first mode seiche decays to  $e^{-1}$  of its initial amplitude in three to four periods. Stravisi (1973) analysed residuals between 31 October and 9 November 1966 from four locations. He assumed a damping factor of the form  $\exp(-Kt/2)$  and estimated damping coefficients  $K$  for the first and second seiche of 1.1 and  $1.7 \times 10^{-5} \text{ s}^{-1}$ , corresponding to decay times of 2.1 and 1.4 d. Robinson *et al.* (1973) analytically approximated the decay of the amplitude of the initial seiche as  $(1 + 5.8et/P)^{-1}$  where  $e = K_D k z_o / h^2$  ( $z_o$  is the initial amplitude of the seiche,  $K_D k = 0.25$  is a non-dimensional drag coefficient,  $h$  is the depth) and  $P$  is the period of the first seiche; the

seiche decays to half amplitude after about  $5P$ , the decay time in that case would equal 4.6 d. They found good agreement with the seiches observed in Venice after the surge on 17 February 1967, when the initial seiche reduces to 10 per cent after 13 periods. Godin and Trotti (1975) modelled six largest seiche events recorded at Trieste between 1952 and 1971 by damped sinusoids from which they found decay times ranging between 3.8 and 5.9 d. Their analysis included the episode from November 1966, and they obtained a decay time of 4.3 d. They also analysed the seiche episode from February 1967, episode 6 in our Table 1. Since a number of smaller wind impulses following the seiche-generating wind event were noticeable in envelopes of filtered residuals, we did not estimate decay time for this episode. The decay time Godin and Trotti obtained was 4.9 d.

## MECHANISMS OF DECAY

### *Bottom friction damping*

Bottom stress is usually specified by a quadratic drag law of the form  $\rho_o C_D u|u|$ , in which  $\rho_o$  is the fluid density,  $C_D$  is a dimensionless drag coefficient the order of 0.001 and  $u$  is the bottom velocity. If the bottom flow is periodic in time, the bottom stress term can be expanded in a Fourier series. Keeping the first term (periodic with flow period) only, the bottom stress that damps the periodic flow becomes a linear function  $k\rho_o u$  of velocity, where  $k$  is a coefficient of bottom friction that depends linearly on the velocity amplitude (e.g. Dean and Dalrymple, 1984). If the bottom flow additionally includes substantial contributions from e.g. surface gravity waves, tides, lower frequency motions, then the bottom stress that damps the periodic flow is still of the form  $k\rho_o u$ , but now  $k$  also depends on the magnitude and spatial distribution of the other flow components. Godin (1988) makes use of the approximation

$$\varepsilon|\varepsilon| \approx 0.5(m\varepsilon + \varepsilon^3/m), m = 0.7, |\varepsilon| < 1 \quad (1)$$

to express  $k$  in the approximate form

$$k(x) = C_D U [0.5(m + 3a_0^2/m + 3a_s^2/4m + (3/2m) \sum a_j^2)] \quad (2)$$

in the case where the velocity  $u$  consists of a mean flow  $u_0(x)$ , a flow of amplitude  $u_s(x)$  harmonic at the seiche frequency  $\sigma_s$ , and tidal and wave-related flows of amplitude  $u_j(x)$  harmonic at frequency  $\sigma_j$ . In (2)

$$U = (u_0^2 + u_s^2 + \sum u_j^2)^{1/2}, a_k = u_k/Uf \text{ or } k = 0, s, j \quad (3)$$

When the bottom stress is of the form  $k\rho_o u$ , both  $k$  and depth  $h$  are constant, and there is no radiation (Otranto is replaced by a node), then it is easily shown that the free decay time  $\tau$  is related to  $k$  by  $\tau = 2h/k$ .

The contribution to damping of bottom stress associated both with the seiche itself and with other motions was estimated using the model with  $k$  given by (2). The model was run starting from rest with an initial sea level profile having approximately the shape of the fundamental seiche. Model sea level time series at Bakar, Split and Dubrovnik were band-pass filtered, envelopes were estimated and represented by exponential curves from which values of decay time for the three locations were estimated and finally averaged. We consider damping associated with (a) bottom velocities associated with the seiche itself, (b) subtidal

bottom velocities associated with wind driven currents, (c) bottom velocities associated with tides, (d) bottom velocities associated with surface gravity waves.

(1) Our model results show that bottom velocities associated with the seiche itself are as great as 15 cm/s in the shallowest part of the Adriatic. If we assume  $C_D = 0.0015$ , the value most modellers use on the shelf for a fine sediment bottom (Grant *et al.*, 1984), our model gives 22.80 d for the self-damping time of a seiche whose amplitude in the far northern Adriatic is 50 cm.

(2) Subtidal bottom currents generated in the northern Adriatic by scirocco winds, from the southeast, are about 10 cm/s (Orlić *et al.* 1994); bora winds, from the northeast, induce bottom velocities which are spatially quite variable and can reach up to 30 cm/s (Orlić *et al.*, 1986). Therefore we estimated typical low frequency wind driven bottom velocities present in the northern Adriatic during moderate winds after the seiche generating wind event ceased, to be 5 cm/s. With  $C_D = 0.0015$  the model with this damping alone gives a decay time of 11.09 d.

(3) Tidal currents in the Adriatic are rather small. Principle axis M2 currents are about 5–10 cm/s (Cavalini, 1985); diurnal currents are even smaller. We represented diurnal tidal currents by diurnal and semidiurnal velocity fields obtained from a two dimensional rectangular model in which the depth was represented by von Sterneck's (1919) discretization and the flow was forced at the open end by incoming Kelvin waves of diurnal and semidiurnal period. The amplitudes of sea level for the three dominant constituents K1, M2 and S2 at the closed northern end of the domain were taken to be 18 cm, 27 cm and 16 cm, the values obtained at Trieste by Godin and Trotti (1975). The decay time obtained by running our model with damping due to these tidal currents alone was 17.67 d. Because tidal frequencies are close to those of seiches, tidal currents can beat with seiches so that tidally induced seiche decay can differ significantly for times when seiches and tides are mostly in or out of phase. Additional variability is present over the spring–neap and declinational cycles. This level of detail was not taken into account in the analysis.

(4) Cavaleri *et al.* (1989) showed that wind waves present *after* a strong scirocco event or after a wind event that starts with bora in the northern Adriatic and scirocco in the rest of the basin and that later turns into scirocco over the whole Adriatic, can be characterized by mean wave period  $T = 6$  s and significant wave height  $H = 2$  m. These parameters give bottom velocities varying from millimeters per second in the deepest water to approximately 20 cm/s in the shallowest part of the basin. Consequently, assuming  $C_D = 0.0015$ ,  $k$  associated with these waves varies from 0 to  $0.3 \times 10^{-3}$  m/s. Since our work is limited to analysing decay of seiches over periods of time when seiche motion is very energetic and the wind contribution is small (after, but not during generation), we assumed in calculating the decay times above, that both the wind driven current and the wind-wave contributions to the overall decay are constant in time, and we estimated them from typical winds observed after seiche-generating wind events in our data. The decay time obtained by running our model with damping due to these waves alone is 105.34 d.

When the model was run with  $C_D = 0.0015$  and all the foregoing flow fields (50 cm seiche, 5 cm/s wind driven currents, tidal currents associated with K1, M2 and S2, wave induced bottom currents associated with 6 s waves of height 2 m) were included in (2), the resulting damping time was 9.46 d. When the seiche amplitude was reduced to 25 cm, the damping time increased only slightly, to 9.96 d. This suggests that the contribution to bottom stress

associated with the seiche itself is much less than the contribution made to bottom stress by the seiche riding on currents of other origins. These effectively linearize the quadratic bottom friction law for the seiche so that while it decays freely we indeed expect to see the linear decay of the logarithm of its amplitude visible on Fig. 5.

The frictional decay time might be shorter than this estimate because our estimates of bottom currents have been made conservatively (although plausibly inflating them does not decrease the estimated frictional decay time by more than about a day), but it might also be shorter because field studies where the bottom stress was estimated directly from bottom boundary layer measurements (Grant *et al.*, 1984) suggest that bottom stresses estimated from  $\rho_0 C_D u |u|$  with  $C_D = 0.0015$  and velocities outside the bottom boundary layer are too small by a factor of two to six. The results of this and similar attempts in the literature to estimate  $C_D$  and  $k$  directly from observations of flows at scales large compared with boundary layer scales are summarized in the Appendix. We conclude that the seiche damping time associated with bottom friction alone may be as long as 9.46 d if  $C_D = 0.0015$ ; the two to six times larger values suggested by the foregoing discussion would result in damping times two to six times shorter. Our ultimate preference is for a value of  $C_D$  that is at least twice the usual value 0.0015, but we do not have a convincing argument that fixes the largest likely value of  $C_D$ . A frictional damping time of 4.73 d, resulting from  $C_D = 0.003$ , corresponds to a spatially constant value of  $k = 0.45 \times 10^{-3}$  m/s in our one dimensional seiche model.

#### *Energy transmission through the open boundary*

We define a coefficient  $a$  of radiation through Otranto by imposing the boundary condition  $\zeta = auh/(gh)^{1/2}$  at the open mouth ( $\zeta$  is sea level,  $g$  the acceleration of gravity). Above, we appealed to models and observations of flow with dynamics different from those of seiches in order to bound the coefficient  $k$  of bottom friction. No such observations are available for estimation of  $a$ .

In order to make an *a priori* estimate of  $a$ , consider a narrow channel along the  $x$  axis with a discontinuity in cross-section at  $x=0$ . The channel has depth  $h$  and width  $b$  for negative values of  $x$ ,  $h'$  and  $b'$  for positive  $x$  values;  $bh$  is smaller than  $b'h'$ . In the case of partial reflection of a plane wave, incident from  $x < 0$ , elevation and velocity for negative  $x$  are given by

$$\zeta = F(t - x/c) + f(t + x/c), \quad u = (g/c)F(t - x/c) - (g/c)f(t + x/c) \quad (4)$$

and for positive  $x$  by

$$\zeta' = \varphi(t - x/c'), \quad u' = (g/c')\varphi(t - x/c') \quad (5)$$

where  $F$  represents the incident wave,  $f$  the reflected wave and  $\varphi$  the transmitted wave, and in which  $c$  and  $c'$  denote the phase speeds  $\sqrt{gh}$  and  $\sqrt{gh'}$  of shallow water waves. It follows from mass conservation that  $uhb = u'h'b'$  for  $x=0$ , while continuity of surface displacement requires  $\zeta = \zeta'$  there. Substitution of (4) and (5) into these gives the ratios of the elevations of the reflected and incident waves, as well as the ratios of elevations of the transmitted and incident waves (Lamb, 1932). From these and (4), the elevation at  $x=0$  can be written as

$$\zeta = auh/(gh)^{1/2} \quad (6)$$

where

$$a = (bc)/(b'c') \quad (7)$$

If there is no discontinuity in cross section of the channel at  $x = 0$ , then  $a = 1$  and (6) becomes the well known relation between elevation and velocity for progressive shallow water waves. By substituting the ratios of elevations of the reflected and incident wave and transmitted and incident wave into the corresponding ratios of the energy of these waves, we have

$$E_{reflected}/E_{incident} = (1 - a)^2/(1 + a)^2, \quad E_{transmitted}/E_{incident} = 4a/(1 + a)^2 \quad (8)$$

These expressions enable us to determine how much of the energy of the incident wave will be reflected and how much will be transmitted through the open boundary. The coefficient  $a$ , which describes the energy transmission, depends only on the geometry of the two basins analysed and falls between 0 and 1.

For shallow depth  $h$  and no bottom friction ( $k = 0$ ) it may be shown that the free decay time  $\tau$  for a gulf connected to a channel is related to  $a$  by

$$\tau = -2L/[C_a(gh)^{1/2}] \quad (9)$$

where

$$C_a = \ln[(1 - a)/(1 + a)] \quad (10)$$

and  $L$  is the length of the basin. This formula (9) applies only in the flat bottom case. It may then be rewritten as  $\tau = -T/(2C_a)$ , in which  $T$  is the seiche period. In this form, it gives an estimate of the order of magnitude of  $a$ ; for a radiative damping time of 6 d,  $a = 0.037$ .

The influence of basin depth on seiche decay time is in the opposite sense in the two cases of bottom friction only ( $a = 0$ ) and radiation only ( $k = 0$ ). In the first case ( $a = 0$ ) the smaller the basin depth, the stronger the influence of bottom friction and consequently, the shorter the decay time. In the second case ( $k = 0$ ), smaller basin depth causes smaller phase velocity; therefore the energy will be removed at a lower rate and the decay time will be longer.

The open mouth condition (6) we applied may be rewritten as

$$\zeta = ubhZ_C \quad (11)$$

where

$$Z_C = 1/(b'c') \quad (12)$$

is a "channel radiation" impedance. This simple condition is physically applicable only if the basin into which the gulf radiates is a channel of width  $b'$  not very different from the width  $b$  of the gulf, so that the radiated field also has negligible cross channel variation. Garrett (1975) has shown that if the gulf debouches into an infinite half plane then the impedance for motion harmonic at radian frequency  $\omega$  is

$$\begin{aligned} Z_O &= (\omega/2gh')\{1 - (2i/\pi)*[\ln(0.5k'b) - 1.5 + \gamma] - (i/\pi)(f/\omega)\ln[(\omega + f)/(\omega - f)]\}, \quad \omega > f \\ Z_O &= (f/2gh')\{1 - (2i/\pi)*(\omega/f)[\ln(0.5k''b) - 1.5 + \gamma] - (i/\pi)\ln[(\omega + f)/(\omega - f)]\}, \quad \omega < f \end{aligned} \quad (13)$$

where  $f$  is the Coriolis parameter,  $h'$  is the depth of the half plane ocean,  $b$  is the channel width, and  $k' = (\omega^2 - f^2)^{1/2}/(gh')^{1/2}$ ,  $k'' = (f^2 - \omega^2)^{1/2}/(gh')^{1/2}$ . The real part of  $Z_O$  quantifies

radiation damping of the seiche in the channel and the imaginary part signals a phase shift of radiation partly reflected back into the channel from the open mouth.

We compare  $Z_C$  and  $Z_O$  for motions periodic with period 21 h in an Adriatic having a width of 100 km at the mouth. Some typical values for  $Z_C(h', b')$  are

- (1)  $Z_C(1000 \text{ m}, 1000 \text{ km}) = 1.01 \times 10^{-8} \text{ s/m}^2$  ( $a = 0.071$ )
- (2)  $Z_C(1000 \text{ m}, 3000 \text{ km}) = 3.37 \times 10^{-9} \text{ s/m}^2$  ( $a = 0.023$ )
- (3)  $Z_C(3000 \text{ m}, 1000 \text{ km}) = 5.83 \times 10^{-9} \text{ s/m}^2$  ( $a = 0.041$ )
- (4)  $Z_C(3000 \text{ m}, 3000 \text{ km}) = 1.94 \times 10^{-9} \text{ s/m}^2$  ( $a = 0.014$ )

The values in parenthesis are  $a = bc/b'c'$  of (7) supposing  $b = 100 \text{ km}$  and  $h = 500 \text{ m}$ . Some typical values for  $\text{Re}[Z_O(h')]$  with  $b = 100 \text{ km}$  and seiche period  $T = 21 \text{ h}$  at latitude  $45^\circ$  are

- (1)  $\text{Re}[Z_O(1000 \text{ m})] = 6.25 \times 10^{-9} \text{ s/m}^2$
- (2)  $\text{Re}[Z_O(3000 \text{ m})] = 2.08 \times 10^{-9} \text{ s/m}^2$

The channel radiation condition (6) gives impedances that are within about half an order of magnitude of the real part of infinite half plane radiation impedances at the seiche frequency for Mediterranean values of the parameters  $h'$  and  $b'$ . We have therefore retained it in subsequent calculations as a simple parametrization of radiative damping. We comment on the larger question of what condition would best represent the Mediterranean, which is neither a channel nor an infinite half plane, in the concluding discussion section.

#### *Interplay of the two mechanisms of decay in the numerical model*

A set of model runs was carried out for different possible values of the bottom friction and radiation coefficients  $k$  and  $a$ :  $k = 0$  to  $1.6 \times 10^{-3} \text{ m/s}$  with an  $0.4 \times 10^{-3} \text{ m/s}$  step,  $a = 0$  to  $0.09$  with an  $0.015$  step. For each case the model was run from an initial sea level profile having approximately the shape of the fundamental seiche and model sea level time series at Bakar, Split and Dubrovnik were analysed in the same manner as the residual sea levels: they were filtered and envelopes were estimated and represented by exponential curves from which values of decay time for the three locations were estimated and finally averaged. In this manner the dependence on  $k$  and  $a$  of the free decay time of the model seiche was delineated. The results are shown in Fig. 7.

Using these results plus the empirically obtained decay time and the bounds on frictional damping time determined for the Adriatic Sea in the beginning of this section, values of  $a$  were estimated as follows.

The one dimensional shallow water model of the seiche damped by bottom stress represented by the quadratic bottom friction law  $\rho_0 C_D u|u|$  using the commonly accepted drag coefficient  $C_D = 0.0015$  and quantitative estimates of bottom currents associated with wind driven currents, tides and wind waves, as well as the seiche itself with no radiation, gives a damping time of 9.46 d. Figure 7 shows that this damping corresponds to a constant value of  $k = 0.22 \times 10^{-3} \text{ m/s}$ . The frictional decay time might be shorter because our estimates of bottom currents have been made conservatively, but plausibly inflating them does not decrease the estimated frictional decay time by more than about a day. From Fig. 7, if  $k = 0.22 \times 10^{-3} \text{ m/s}$ , the radiation needed to decrease the damping time to the observed 3.2 d requires  $a = 0.043$ . The radiation then accounts for 66% of the energy loss per period. But independent estimates of bottom friction for Adriatic wind driven currents and inertial

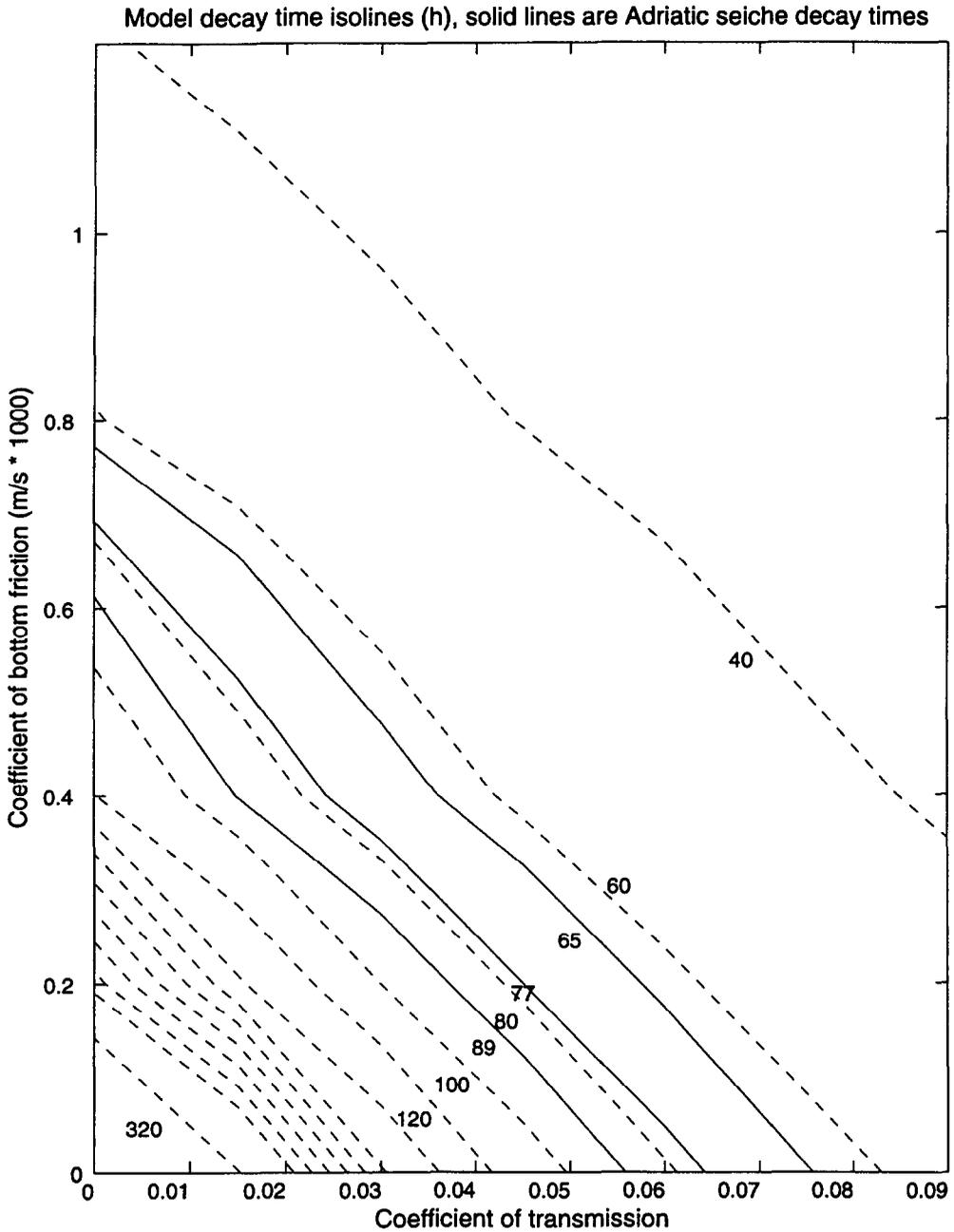


Fig. 7. Contours (dashed, in hours) of average decay time obtained from the numerical model incorporating linear bottom friction (parameterized by  $k$  in the bottom stress term  $k\varrho_0u$ ) and transmission through Otranto (parameterized by  $a$  in (6)). Also drawn are contours (solid) for the empirically obtained decay time  $3.2 \pm 0.5$  d.

oscillations, as well as comparisons between quadratic law bottom stress and directly measured bottom stress, all suggest that the quadratic law with  $C_D = 0.0015$  substantially underestimates the bottom stress. Based on these studies, a more appropriate value of the drag coefficient is at least  $C_D = 0.003$ . In this case, the one dimensional shallow water model of the seiche damped by bottom stress represented by the quadratic bottom friction law gives a damping time of 4.73 d. Figure 7 shows that this damping corresponds to a constant value of  $k = 0.45 \times 10^{-3}$  m/s. From Fig. 7, if  $k = 0.45 \times 10^{-3}$  m/s, the radiation needed to decrease the damping time to the observed 3.2 d requires  $a = 0.011$ . The radiation then accounts for 32% of the energy loss per period. The corresponding figures for  $C_D = 0.004$  are 3.55 d,  $k = 0.64 \times 10^{-3}$  m/s, and  $a = 0.005$  so that radiation accounts for about 9% of the energy loss per period. But we have no convincing reason to prefer  $C_D = 0.004$  over  $C_D = 0.003$ .

## DISCUSSION

The principle conclusions are summarized in the abstract; this section comments on certain aspects of the analysis.

Model experiments showed that, to obtain an estimate of free decay time accurate to a few percent, a period of free decay lasting five to ten periods would be necessary. Most of our seiche events were of sufficiently small amplitude that wind influence on seiche decay was noticeable after only a few periods, so that empirical decay time estimates are in appreciably greater error.

The effect of across-basin winds on seiche decay has already been noted above (Fig. 6) for episode 4. Episode 3 is the one with the strongest across-basin winds, and the only one in which strong across-basin winds immediately follow the initial along-basin wind pulse. During the second half of 2 December and the beginning of 3 December 1964, a strong bora was blowing from the NE in the northern Adriatic and a strong scirocco from the SE over the rest of the basin. During 3 December the bora ceased. The next day the scirocco turned into very strong across-basin winds above the middle Adriatic and weaker across-basin winds above the southern part of the basin, indicating the occurrence of significant along-basin variability of across-basin winds. Comparison of observed elevations and elevations modelled with these winds suggests there is significant influence of along-basin-variable transverse winds on longitudinal seiches.

A possible mechanism for this is suggested by the observation of Finizio *et al.* (1972), who note that "winds blowing from the Istrian peninsula across the northern Adriatic may give a contribution to the sea level rise at Venice". If across-basin winds vary appreciably along the Adriatic, so will such an initial setup; when the winds relax it may thus radiate strongly into along-basin-propagating Kelvin waves. A two dimensional model would be necessary to verify this speculation.

Sea level fluctuations induced by changes in atmospheric pressure frequently accompany seiches (Franco *et al.*, 1982), but they are at periods of several days; they do not influence our decay time estimates because these were made from data band-passed around the seiche period.

Several authors have shown that a one dimensional model can very well describe the period and structure of the lowest modes of an elongated basin (Schwab and Rao, 1983). Thus, for example, the fundamental mode of the two dimensional numerical model of the Adriatic constructed by Michelato *et al.* (1985) shows very little across-basin variation. Two

dimensional model results of Accerboni *et al.* (1973) and of Stravisi (1973) similarly support the adequacy of a one dimensional model for description of the fundamental Adriatic seiche mode.

The Adriatic seiche is well approximated as the quarter-wave resonance of a channel with a node at the open end. The Mediterranean with a node across the mouth of the Adriatic would have a mode whose frequency might or might not be close to that of the seiche. If it were close, then these two modes could be viewed as gently coupled harmonic oscillators (gently coupled because the mouth width is small; in consonance with the small size of Garrett's (1975) "half plane admittance"). The closer their frequencies ( $\omega_O$  and  $\omega_C$ ) the more slowly they would exchange energy. Thus even with no dissipation in either basin, the seiche would initially decay at a rate determined by the infinite half plane admittance. After reflected waves arrived it would ultimately grow again to its initial size after a time of the order of  $1/(\omega_O - \omega_C)$ . Numerical experiments with the extended model support this suggestion. Unfortunately we do not have sufficient information about the normal modes of the Mediterranean to estimate the time scale  $1/(\omega_O - \omega_C)$ .

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## APPENDIX

By measuring bottom boundary layer velocities and using them to directly estimate bottom stress during the CODE experiments, Grant *et al.* (1984) found that mean stress values are three to seven times larger than those obtained using one meter velocities in  $\rho_0 C_D u|u|$  with  $C_D = 0.0015$ . The bottom currents used above in (2) were on the basis of current meter observations 3 m above the bottom for wind driven currents (Orlić *et al.*, 1986) and from a shallow water model for the seiche and the tides. They are thus probably somewhat greater than one meter velocities, in as much as the thickness of the turbulent Ekman layer which overlies the logarithmic layer is probably substantially greater than one meter, so that the factor of three to seven quoted above may be too large for the bottom velocities we have estimated. Orlić *et al.* (1986) estimate that at a station in the northern Adriatic, the wind driven flow at instruments 3 m above the bottom are 1.1 to 1.2 times currents 1 m above the bottom; the factor of three to seven quoted above thus might be two to six. We conclude that by using  $C_D = 0.0015$  we may have underestimated damping due to bottom friction, possibly by a factor of two to six. The decay time of 9.46 d, resulting from the inclusion of all processes important in linearizing the bottom drag and from assuming  $C_D = 0.0015$ , corresponds to a spatially constant value of  $k = 0.22 \times 10^{-3}$  m/s in our one dimensional seiche model. This value is at least half an order of magnitude smaller than local values of  $k$  estimated by analysing other physical processes in the northern Adriatic. Thus Orlić (1987) estimated  $0.6 \times 10^{-3}$  m/s  $< k < 1.5 \times 10^{-3}$  m/s independently by analysing inertial oscillations recorded at a station in the northern part of the Adriatic, and Orlić *et al.* (1986) estimated  $1.13 \times 10^{-3}$  m/s  $< k < 1.24 \times 10^{-3}$  m/s by balancing surface wind stress and bottom stress associated with subtidal flows at a station in the far northern Adriatic. These last local values of  $k$  of Orlić *et al.* (1986) were estimated for a period of time when strong bora was blowing with typical wind speeds the order of 12 m/s. In the observations and in the three dimensional wind driven current model of Orlić *et al.* (1994), a typical wind impulse of 10–12 m/s induced bottom currents of about 30 cm/s. Under such winds, the wave field is characterized by a significant wave height of 2 m and a wave period of about 6 s; the resulting orbital velocities at the depth (28 m) of the station where these observations were made are about 0.11 m/s. Retaining the above estimates (0.10 m/s) for tides and assuming that the seiche was not significantly excited during these observations gives a local coefficient  $k$  of linearized bottom friction that is, in the notation of (2),  $k = C_D U [0.5(ma_0 + a_0^3/m + (3a_0/2m) \sum a_j^2)] = 0.56 \times 10^{-3}$  m/s provided  $C_D = 0.0015$  (Godin, 1988). By matching surface wind stress and bottom stress calculated from observations of the wind driven current only at a station in the northern Adriatic, Orlić *et al.* (1986) estimated  $k = 1.1$  to  $1.2 \times 10^{-3}$  m/s. These values are then consonant with the finding of Grant *et al.* (1984) that  $C_D = 0.0015$  may be too small, although only by a factor of about two. Correspondingly, Orlić *et al.* (1986) balanced surface wind stress and bottom stress using the quadratic law and  $4.67 \times 10^{-3} < C_D < 6.12 \times 10^{-3}$ . Similar values for  $C_D$  were found by Winant and Beardsley (1979), who balanced surface wind stress and bottom stress at a number of locations on the US east coast and off southern California, and by Lentz (1984), who constructed an alongshore momentum balance that also included the pressure gradient and the local acceleration off southern California.