Eliassen-Palm Cross Sections
Edmon *et al.* (1980)

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Eliassen-Palm Flux

For

- β-plane
- Coordinates (y, p) in northward, vertical directions
- Zonal means

$$\mathbf{F} = \begin{pmatrix} -\overline{v'u'} \\ \\ f(y)\dfrac{\overline{v'\theta'}}{\overline{\theta}_p} \end{pmatrix}$$

$\nabla \cdot \mathbf{F}$ will provide a diagnostic for the eddy-induced forcing of the mean state.

Transformed QG Equations

Neglect ageostrophic terms

$$\begin{aligned}
\frac{\partial \overline{u}}{\partial t} - f\,\overline{v}^* - \overline{\mathfrak{F}} &= \nabla \cdot \mathbf{F} \\
f\,\overline{u}_p - R\,\overline{\theta}_y &= 0 \\
\overline{v}^*_y + \overline{\omega}^*_p &= 0 \\
\frac{\partial \overline{\theta}}{\partial t} + \overline{\theta}_p\,\overline{\omega}^* - \overline{\mathfrak{Q}} &= 0
\end{aligned}$$

$\overline{\mathfrak{F}}, \overline{\mathfrak{Q}}$ are Eulerian-mean friction and heating
R is the gas constant times $(p_0/p)^{1/\gamma} p_0$, γ is specific heat ratio

$\nabla \cdot \mathbf{F}$ is the ONLY internal forcing to mean state by disturbances, comprising the total effect of QG eddies, regardless of other properties (nonlinear, transient, turbulent)

Transformed Velocities

$$\begin{aligned}\overline{v}^* &= \overline{v} - \frac{\partial(\overline{v'\theta'}/\overline{\theta}_p)}{\partial p} \\ \overline{\omega}^* &= \overline{\omega} + \frac{\partial(\overline{v'\theta'}/\overline{\theta}_p)}{\partial y}\end{aligned}$$

"Residual meridional circulation" associated with adiabatic processes

Includes heat and momentum transfer that results from interactions of the disturbances

Residual Circulation Streamfunction

$$
\begin{aligned}
f\,\overline{u}_p - R\,\overline{\theta}_y &= 0 \\
\frac{\partial \overline{u}}{\partial t} - f\,\overline{v}^* - \overline{\mathfrak{F}} &= \nabla \cdot \mathbf{F} \\
\frac{\partial \overline{\theta}}{\partial t} + \overline{\theta}_p\,\overline{\omega}^* - \overline{\mathfrak{Q}} &= 0
\end{aligned}
$$

These equations can be combined to eliminate t dependence, and if we introduce the streamfunction $\overline{\psi}^*$ the result is

$$
f^2\,\overline{\psi}^*_{pp} + R\left(\left|\overline{\theta}_p\right|\overline{\psi}^*_y\right)_y = -f\,(\nabla \cdot \mathbf{F})_p - f\,\overline{\mathfrak{F}}_p + R\,\overline{\mathfrak{Q}}_y
$$

Note that $\overline{\psi}^*$ depends on time, but equation has no time derivatives!

Mean Circulation

Holton (1992)

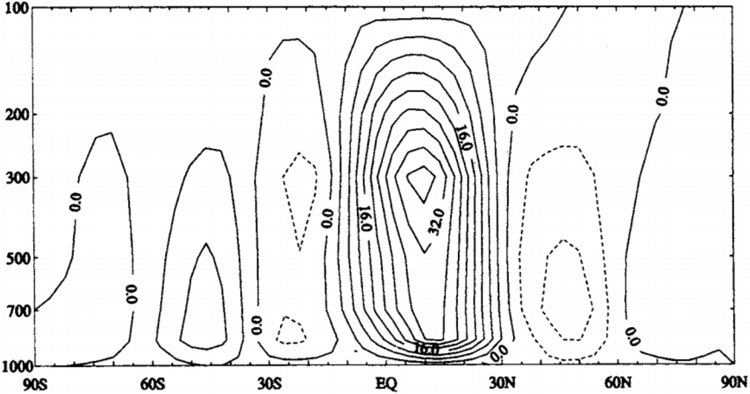


Fig. 10.7 Streamfunction (units: 10^2 kg m^{-1}s^{-1}) for the observed Eulerian mean meridional circulation for Northern Hemisphere winter, based on the data of Schubert et al. (1990).

Right of main cell, circulation appears as though heat travels from higher to lower latitudes – not physical!

Residual Circulation

Holton (1992)

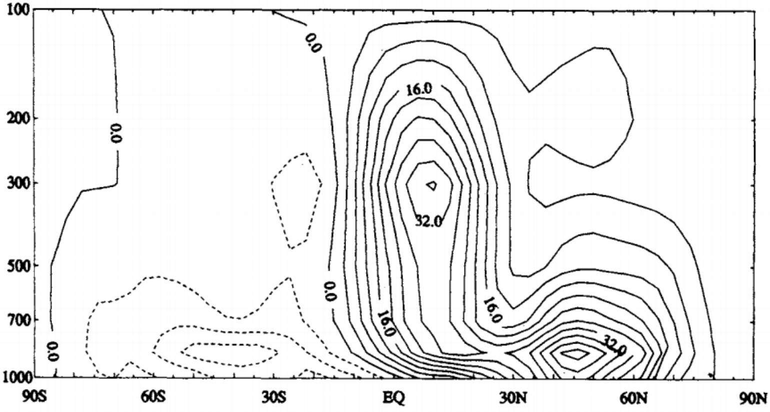


Fig. 10.9 Residual mean meridional stream function (units: 10^2 kg m^{-1} s^{-1}) for Northern Hemisphere winter, based on the data of Schubert et al. (1990).

Residual circulation shows us how heat and momentum are physically transferred

Theorems

The previous equations hold for QG assumptions. The Eliassen-Palm theorem (1961) was that, for steady, conservative, wavelike disturbances, $\nabla \cdot \mathbf{F} = 0$.

Charney and Drazin's "nonacceleration theorem" (1961) noted that if $\nabla \cdot \mathbf{F}, \overline{\mathfrak{F}}, \overline{\mathfrak{Q}}$ are all zero, there is a steady state solution where $\overline{u}_t, \overline{\theta}_t, \overline{v}^*, \overline{\omega}^* = 0$.

The equations show both theorems, but do not depend on how closely the conditions are met.

Relationship to QG Potential Vorticity

Alternatively, the diagnostic can be written as

$$\nabla \cdot \mathbf{F} = \overline{v'q'}$$

for

$$q' = v'_x - u'_y + f\ \left(\theta'/\overline{\theta}_p\right)_p$$

The sign of $\nabla \cdot \mathbf{F}$ can then be given by the opposite of the QG PV flux

$$\overline{q}_y = \beta - \overline{u}_{yy} + f\ \left(\overline{\theta}_y/\overline{\theta}_p\right)_p$$

Thus $\nabla \cdot \mathbf{F}$ is the northward eddy flux of QG PV

Nonlinear Case

Even for nonlinear QG, under nonacceleration conditions

$$0 = q_t + uq_x + vq_y$$

With horizontal incompressibility, this is

$$0 = u_t + (uq)_x + (vq)_y$$

Finally, taking zonal means and using definitions of means and disturbances

$$\overline{q}_t + (\overline{v'q'})_y = 0$$

Thus, if $\overline{v'q'}$ vanishes somewhere, then

$$\overline{v'q'} = \nabla \cdot \mathbf{F} = 0$$

Boundary condition of $\overline{v'q'}$ can be avoided with alternative hypotheses

Wave Theory

Consider a conservation equation

$$\frac{\partial A}{\partial t} + \nabla \cdot \mathbf{F} = D$$

For conservative motion, $D = 0$

A can be called the "EP wave activity," and in QG is approximately

$$A \approx \frac{1}{2}\frac{\overline{q'^2}}{\overline{q}_y}$$

for weak dissipation and nonlinearity

If $\overline{q}_y$ vanishes somewhere, this should instead be the disturbance-associated northward displacement

$$A \approx \frac{1}{2}\overline{q}_y\overline{\eta'^2}$$

Benefit of **F**

The conservation equation is not unique, but **F** has two properties that an arbitrary vector might not have:

- For planetary waves of small latitudinal and vertical wavelength, the group-velocity concept is applicable (Lighthill 1978) and it can be shown that

$$\mathbf{F} = \mathbf{c}A$$

  where **c** is group velocity projected onto meridional plane

- **F** appears to be the most convenient choice to compute from observations

Direction of **F**

Thus **F** represents the net wave activity propagation from one height and latitude to another

Even when wave propagation is not valid, tilt of arrows still compares the relative magnitude of eddy heat and momentum fluxes

Spherical Geometry

Real atmospheric data is given in degrees latitude ϕ, so we require a conversion to spherical coordinates.

$$\mathbf{F} = \begin{pmatrix} -r_0 \cos\phi \overline{v'u'} \\ \\ f\, r_0 \cos\phi \dfrac{\overline{v'\theta'}}{\overline{\theta}_p} \end{pmatrix}$$

for the radius of the Earth r_0 and

$$f = 2\Omega \sin\phi$$

for the Earth's angular velocity Ω

Spherical Geometry

$$\nabla \cdot \mathbf{F} = (r_0 \cos\phi)\overline{v'q'}$$

$$q' = \frac{v'_\lambda - (u' \cos\phi)_\phi}{r_0 \cos\phi} + f\ \left(\theta'/\overline{\theta}_p\right)_p$$

for longitude λ

$$\overline{q}_\phi = 2\Omega\cos\phi - \left[\frac{(\overline{u}\cos\phi)_\phi}{r_0\cos\phi}\right]_\phi + f\ \left(\overline{\theta}_\phi/\overline{\theta}_p\right)_p$$

True derivative only in QG approximation

Data

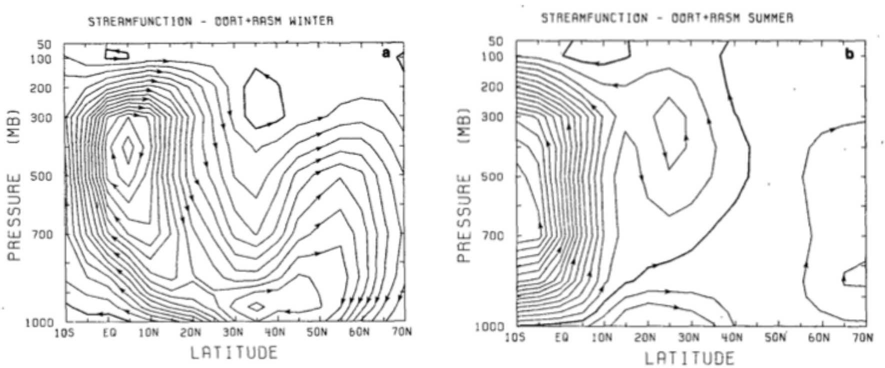
Oort and Rasmusson (1971)

- Eddy and time-mean statistics for June 1958-May 1963
- Winter: December-February
- Summer: June-August
- Pressure levels 1000, 950, 900, 850, 700, 500, 400, 300, 200, 100, 50 mb
- Latitudes 10°S-75°N in 5° increments

National Meteorological Center 11-year average

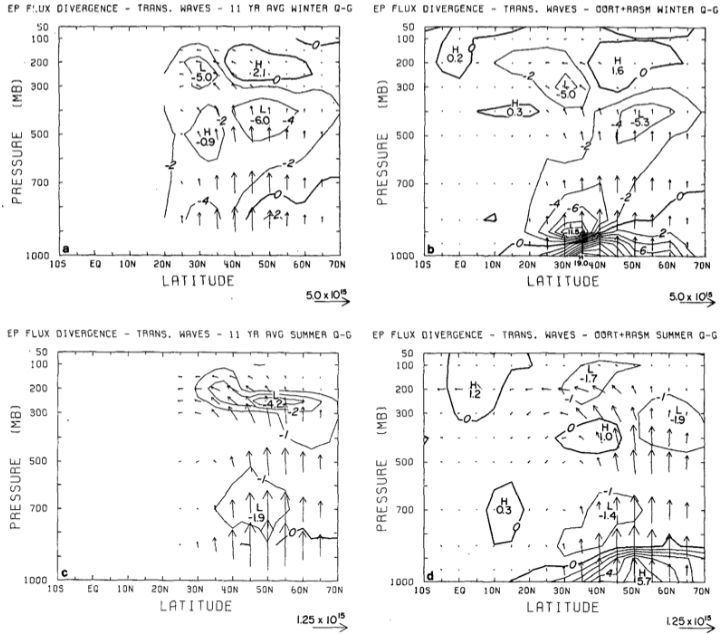
- Twice daily analysis for 1965-1977
- Winter: 120-day period from 15 November
- Summer: 120-day period from 1 June
- Grid of 2.5° latitude by 5° longitude grid starting at 20°N

Hadley and Ferrel Cells

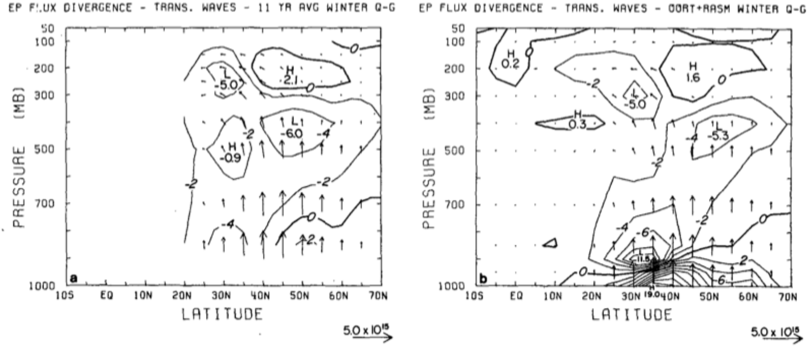


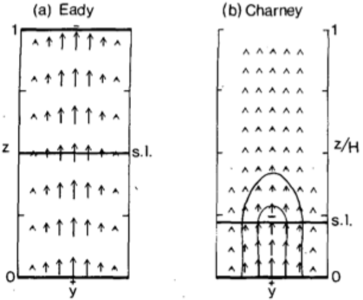
Looks similar to Holton

Contribution of Transient Eddies



Compare to Models





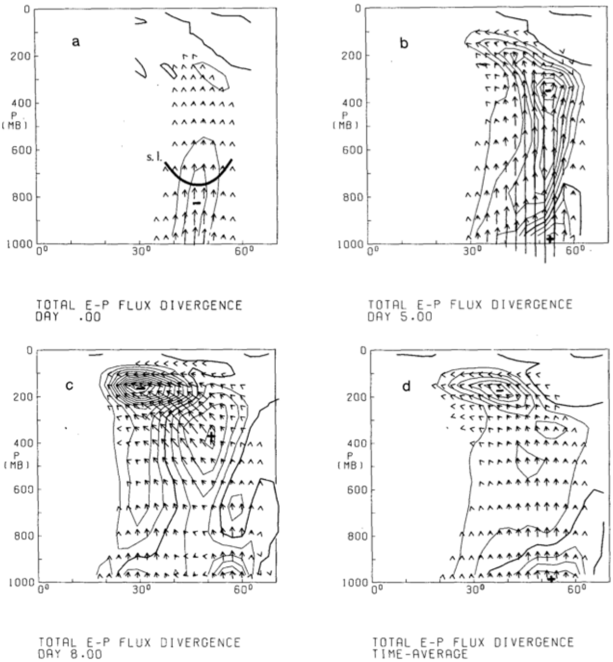
Little resemblance

Simulation

Most unstable baroclinic wave disturbance to a jet: zonal wavelength 6

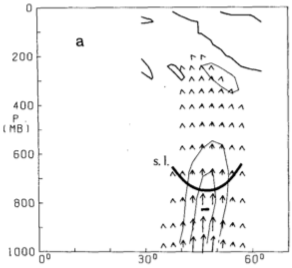
Linear

Nonlinear/
Time-average

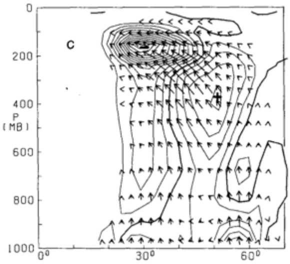


Life Cycle Physics

$$\mathbf{F} = \begin{pmatrix} -r_0 \cos\phi\, \overline{v'u'} \\ \\ f\, r_0 \cos\phi\, \dfrac{\overline{v'\theta'}}{\overline{\theta}_p} \end{pmatrix}$$

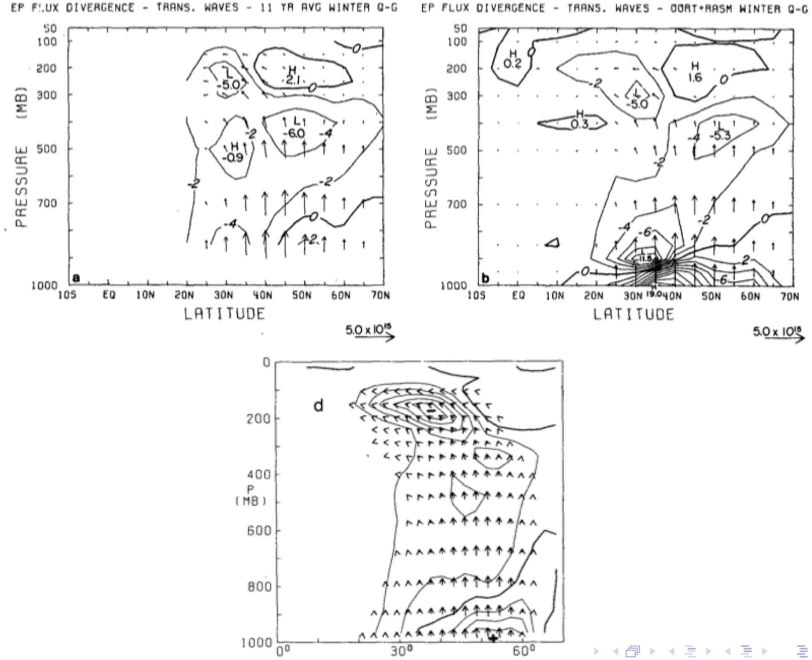


TOTAL E-P FLUX DIVERGENCE
DAY .00

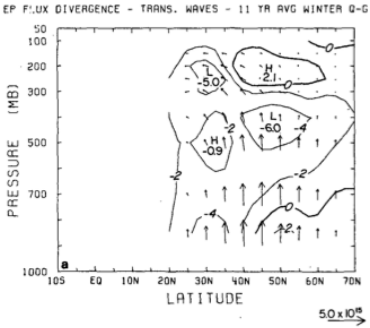


TOTAL E-P FLUX DIVERGENCE
DAY 8.00

Comparison


EP FLUX DIVERGENCE - TRANS. WAVES - 11 YR AVG WINTER Q-G
EP FLUX DIVERGENCE - TRANS. WAVES - OORT+RASM WINTER Q-G
PRESSURE (MB)
LATITUDE
5.0 x 10^15
P (MB)
a
b
d

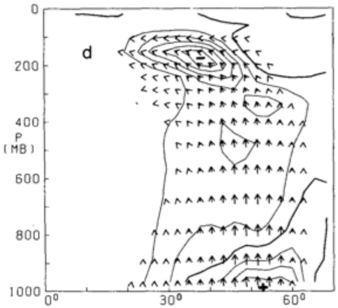
QG PV flux



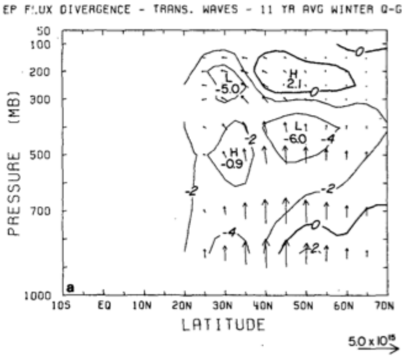
- Net positive $\overline{v'q'}$ at bottom

- $\overline{q}_\phi$ only negative from 60-74°

- Friction and diabatic effects must have influence



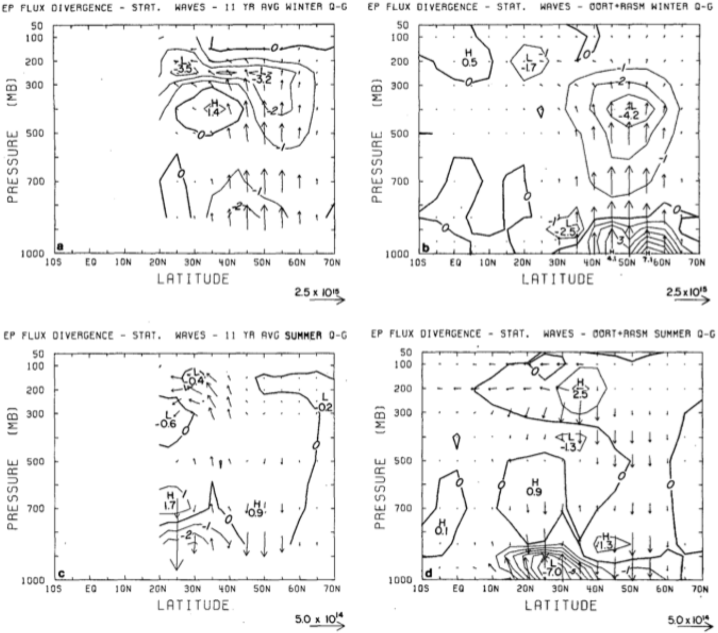
QG PV flux



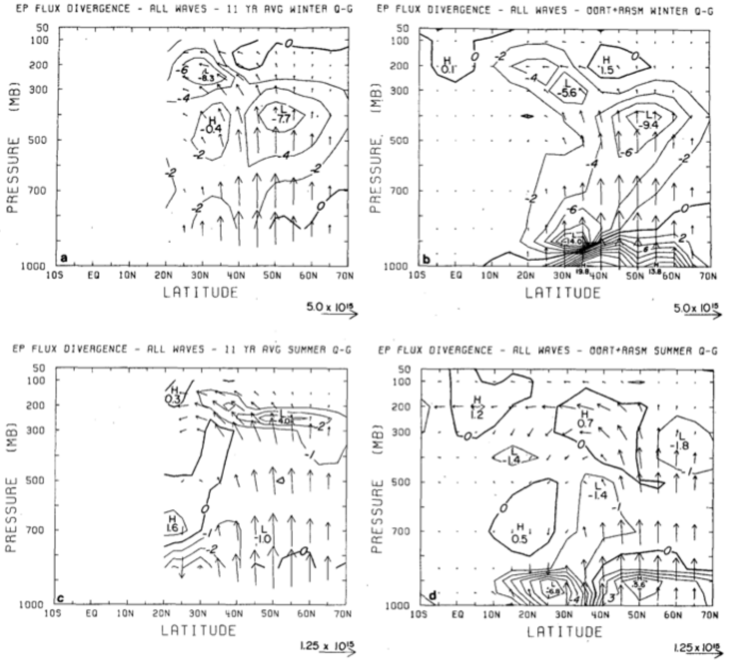
Positive $\nabla \cdot \mathbf{F}$ at 50°, 200 mb

"Likely to be indicative of negative $\overline{q}_\phi$ [. . .] consistent with the fact that stationary disturbances cannot by themselves cause irreversible air-parcel dispersion"

Contribution of Stationary Eddies



Total Contribution



Conclusions

- Diagnostics $\mathbf{F}$ and $\nabla \cdot \mathbf{F}$ provide insight into the physics of eddy/mean flow interaction using Eulerian statistics
- Do not require restrictive assumptions (EP or Charney & Drazin theorems)
- Transient eddy contribution resembles baroclinic instability lifecyce
- Stationary eddies do not seem statistically robust, no model to compare