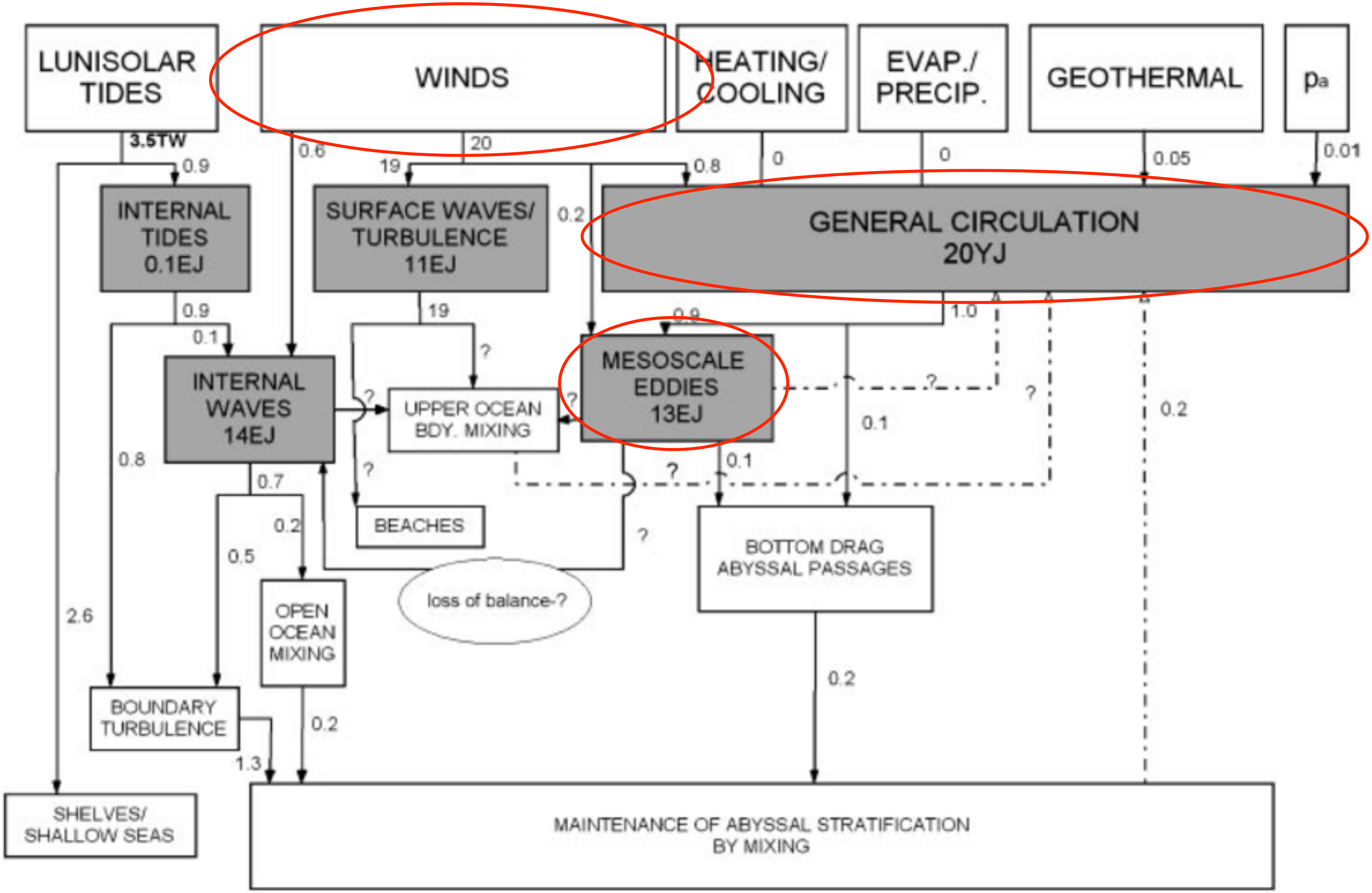
Energy partition in the large-scale ocean circulation and the production of mid-ocean eddies

by Gill, Green and Simmons

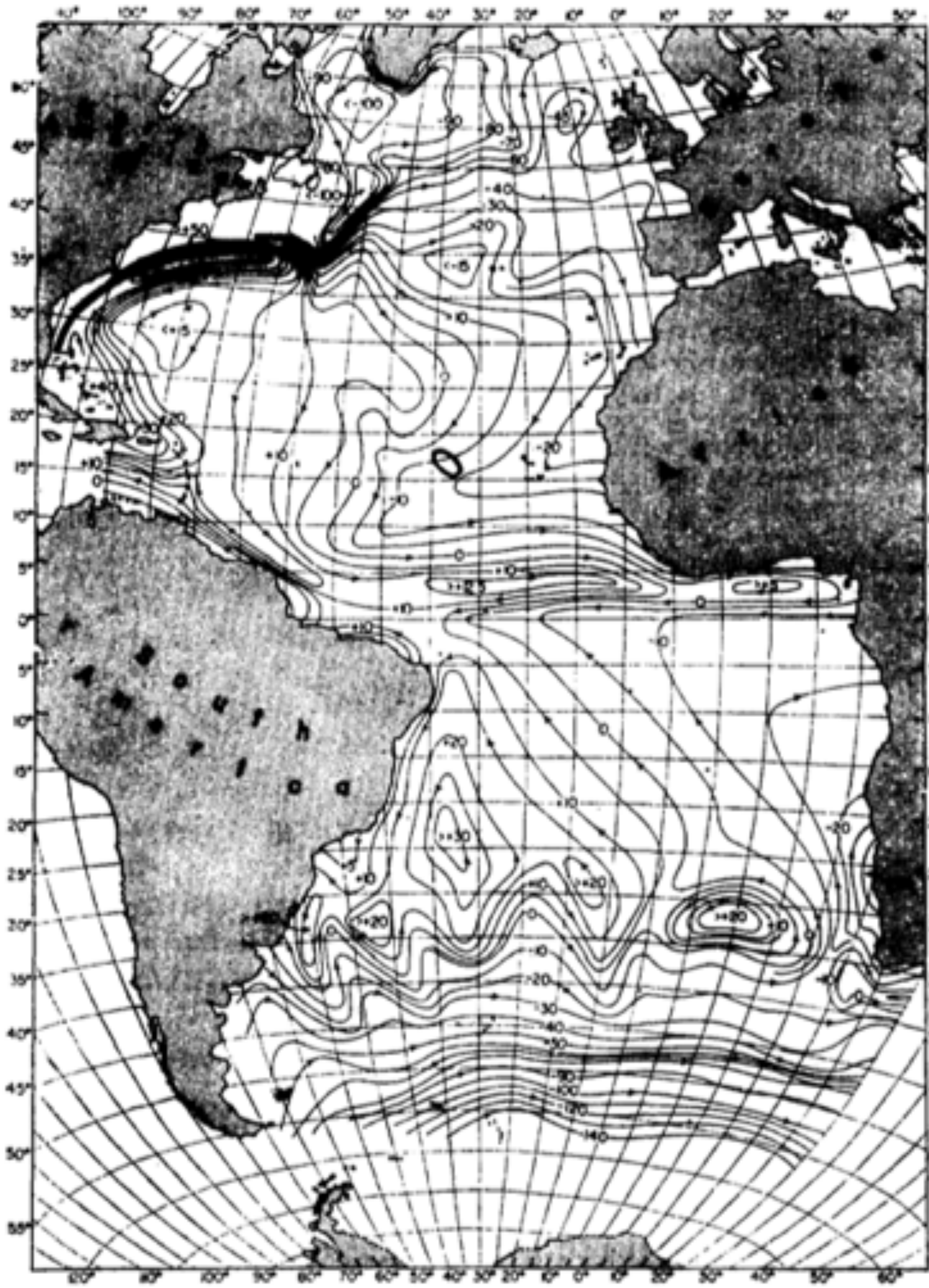
Catherine Jones

Energy in the ocean

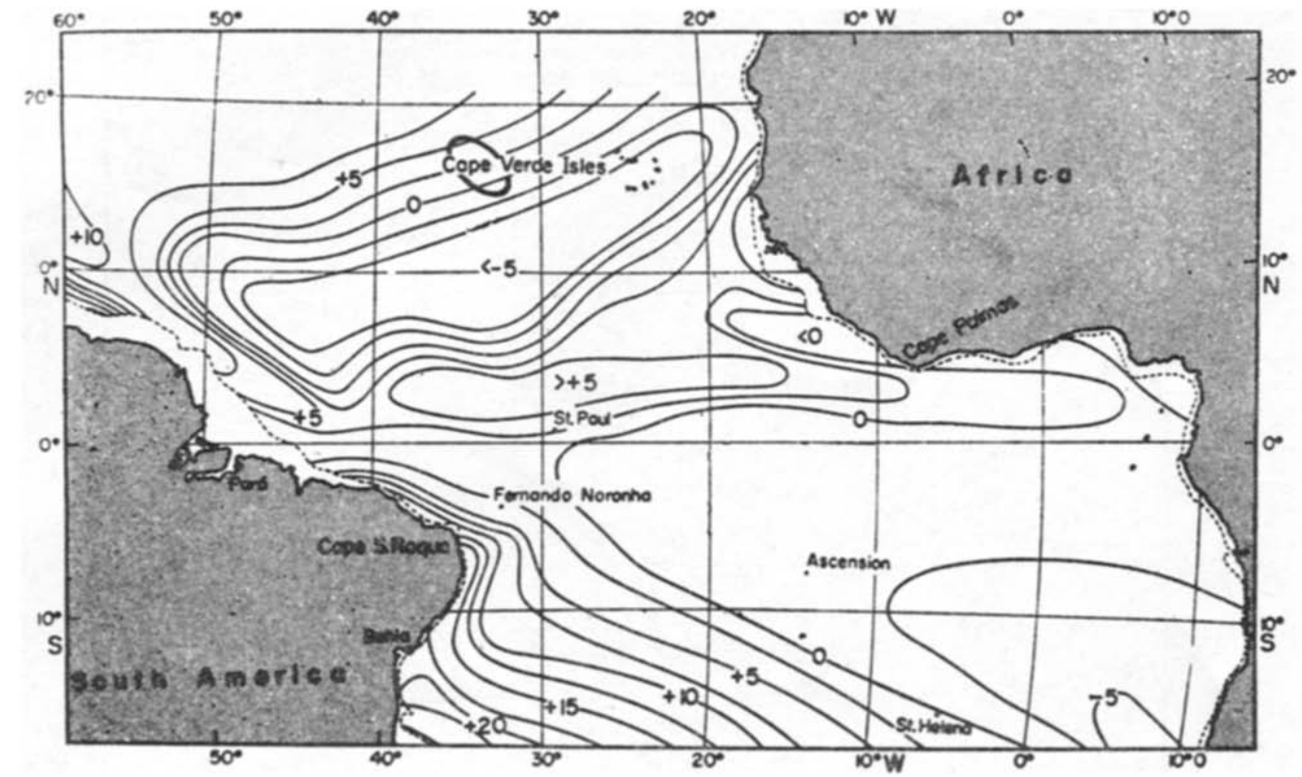


Wunsch & Ferrari (2004)

Observations of eddies



Dynamic topography at the ocean surface



Dynamic topography at 100db

Eddy from POLYGON experiment is superimposed

Eddies, with transient velocities unrelated to the mean flow were a new thing when this paper came out

Observations of eddies

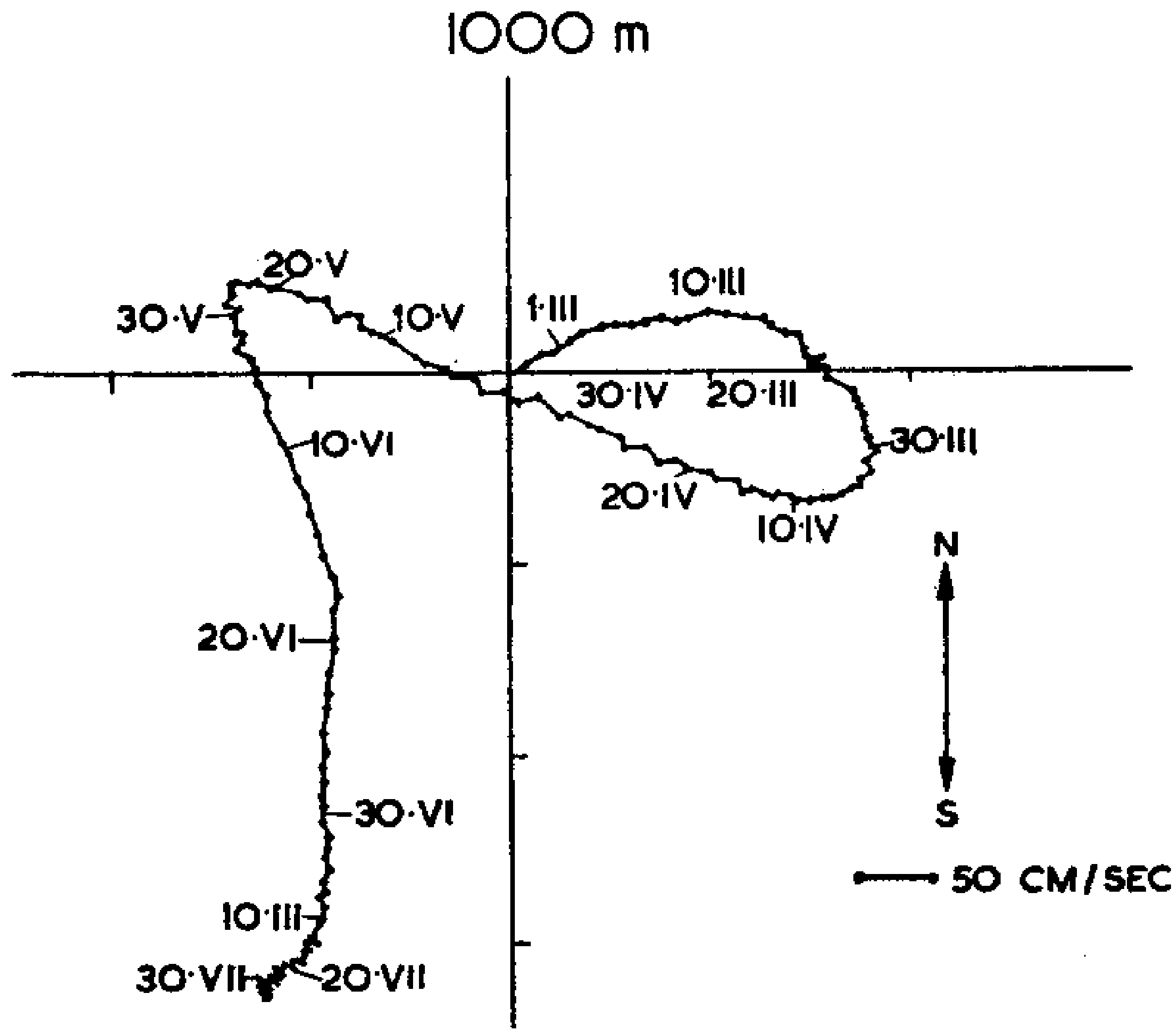
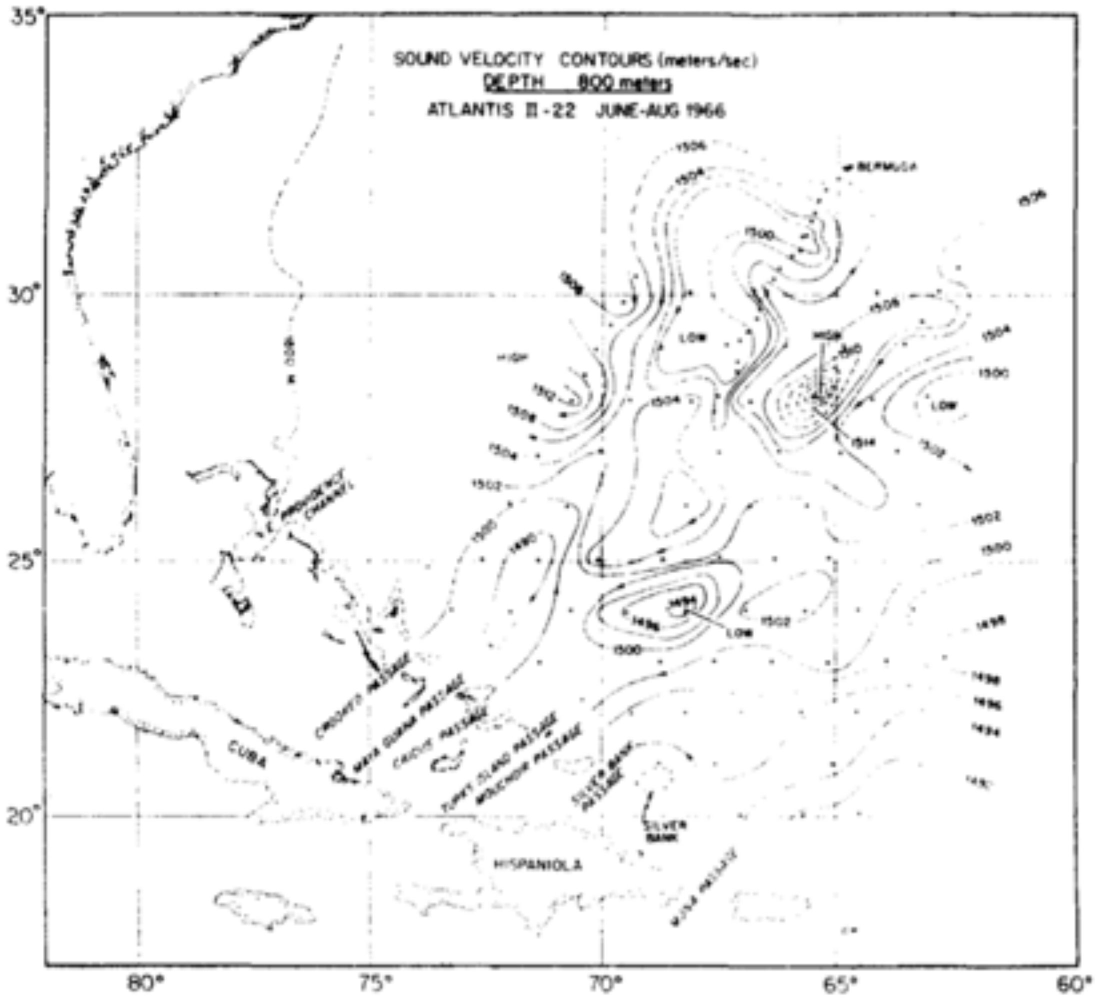
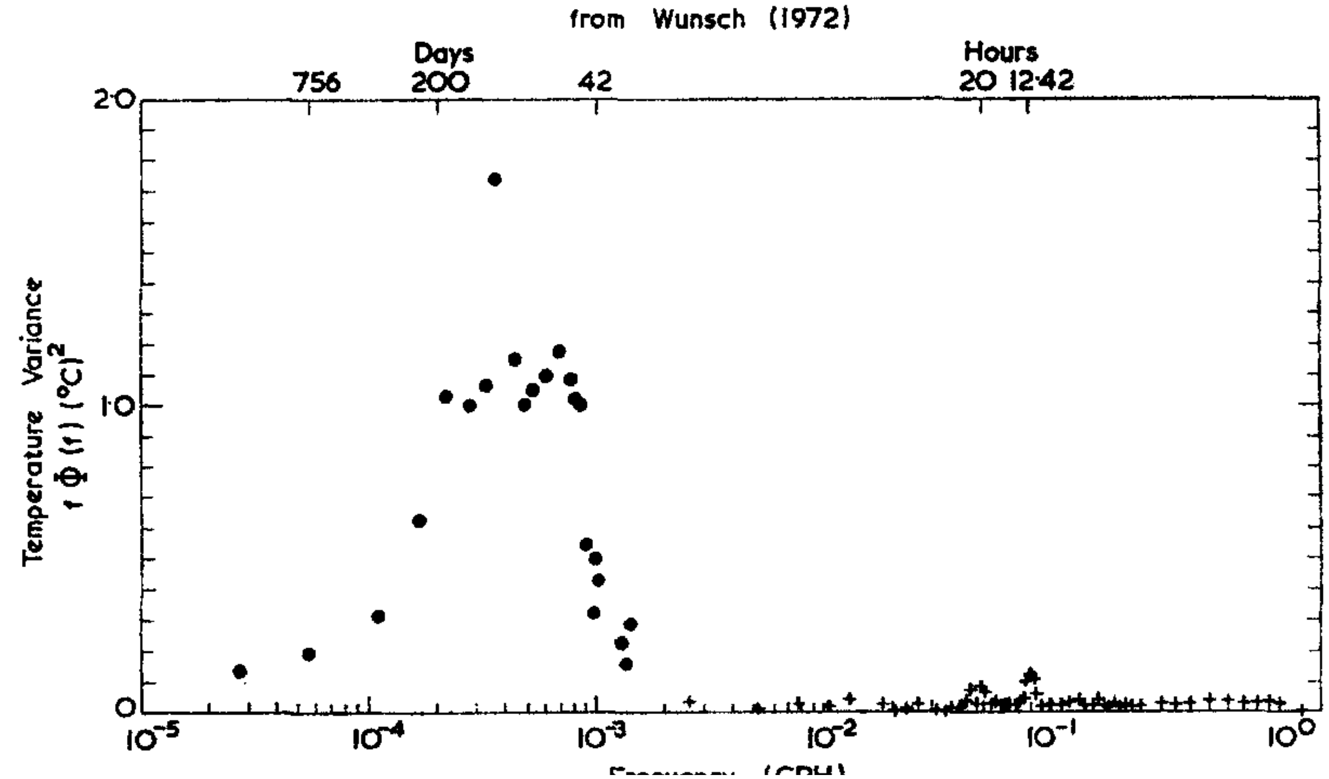


Fig. 2. A progressive vector diagram of currents at 1000 m obtained in the POLYGON experiment (from Brekhovskikh, Federov, Fomin, Koshlyakov and Yampolsky, 1971, Fig. 4).

Observations of eddies



Sound velocity at 800m in the Sargasso Sea (Beckerle, 1972)



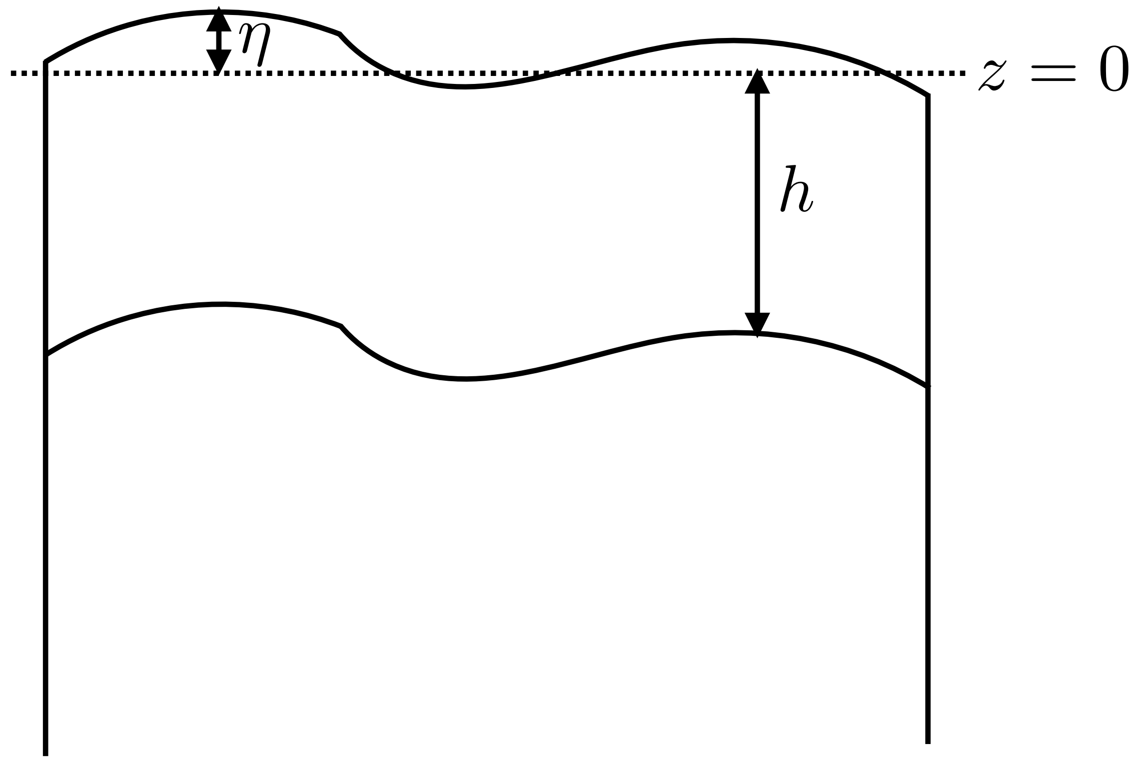
Temperature spectrum: most variance is at timescales of 40 to 100 days (Wunsch,1972b)

How is energy partitioned between the eddies and the mean flow?

Is there enough energy in the mean flow to drive the eddies?

Are the eddies generated locally or at a distance?

1.5 layers



$$g\eta = F_1(z) + g'h$$

Pressure

$$p = \rho_1 g(\eta - z) \qquad -h < z < \eta$$

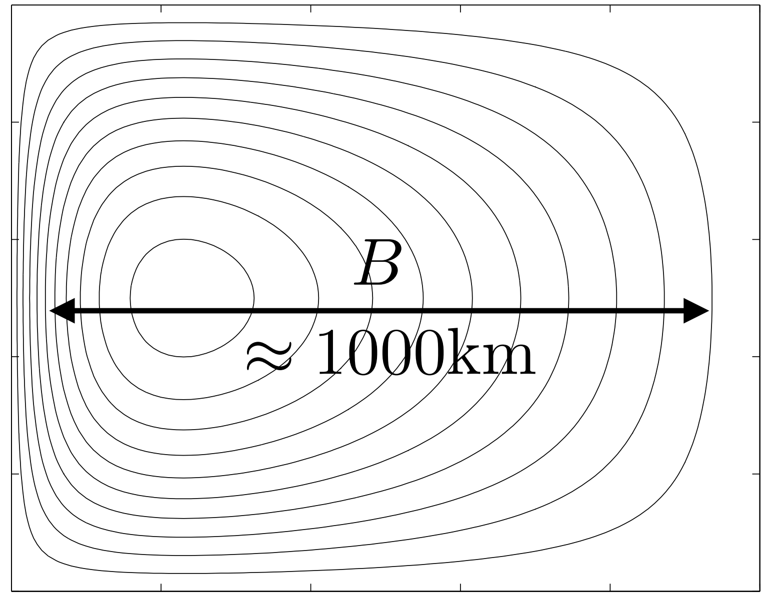
$$p = \rho_1 g(\eta + h) - \rho_2 g(h + z) \qquad z < -h$$

Geostrophy

$$-fv = \frac{-p_x}{\rho_1} = -g'h_x$$

$$fu = \frac{-p_y}{\rho_1} = -g'h_y$$

Energy partition in a gyre



Internal radius of deformation

$$a = \frac{g' h^{\frac{1}{2}}}{f} \approx 30\text{km}$$

$$\overline{\text{KE}} = \frac{1}{2}\rho_1 \overline{h(u^2+v^2)} = \frac{1}{2}\rho_1 g' \overline{h(h_x^2+h_y^2)/f^2}$$

$$\overline{\text{APE}} = \frac{1}{2}\rho_1 g' (\overline{h^2} - (\overline{h})^2)$$

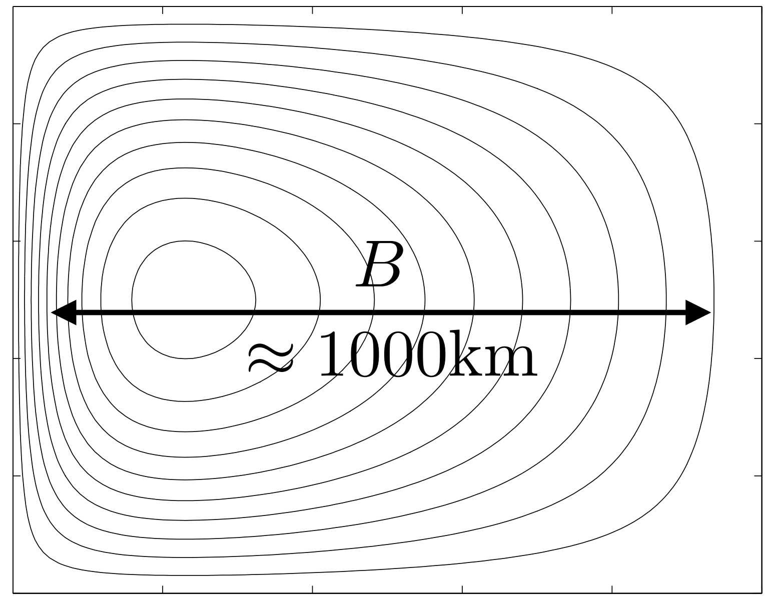
$$h = \overline{h} + h'$$

$$\overline{\text{KE}} \sim \frac{\rho_1 g'^2 \overline{h}\,\overline{h'^2}}{B^2 f^2}$$

$$\overline{\text{APE}} \sim \rho_1 g'^2 \overline{h'^2}$$

$$\frac{\overline{\text{APE}}}{\overline{\text{KE}}} \sim \frac{B^2}{a^2} \approx \left(\frac{1000\text{km}}{30\text{km}}\right)^2 \approx 1000$$

Energy partition in a gyre



Internal radius of deformation

$$a = \frac{g' h^{\frac{1}{2}}}{f} \approx 30\text{km}$$

$$E = \frac{1}{2}\left(\rho_0\left(\psi_x^2 + \frac{f^2\psi_z^2}{N^2}\right)\right)$$ QG expression for energy

$$\overline{\text{KE}} \sim \frac{\rho_0\psi^2}{B^2}$$

$$\overline{\text{APE}} \sim \frac{\rho_0 f^2\psi^2}{N^2H^2} \sim \frac{\psi^2\rho_0}{a^2}$$

$$\frac{\overline{\text{APE}}}{\overline{\text{KE}}} \sim \frac{B^2}{a^2} \approx \left(\frac{1000\text{km}}{30\text{km}}\right)^2 \approx 1000$$

Energy input by the wind

Continuity

$$(hu)_x + (hv)_y + w_{Ek} = 0$$

Geostrophy

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$$-\rho_1 g'(uh^2)_x - \rho_1 g'(vh^2)_y - \rho_1 g' h w_{Ek} = 0$$

Integrate

$$-\rho_1 g' \left[\int uh^2 \, \mathrm{d}y\right]_{x=0} - \rho_1 g' \int\int h w_{Ek} \, \mathrm{d}x\mathrm{d}y = 0$$

output ???

input

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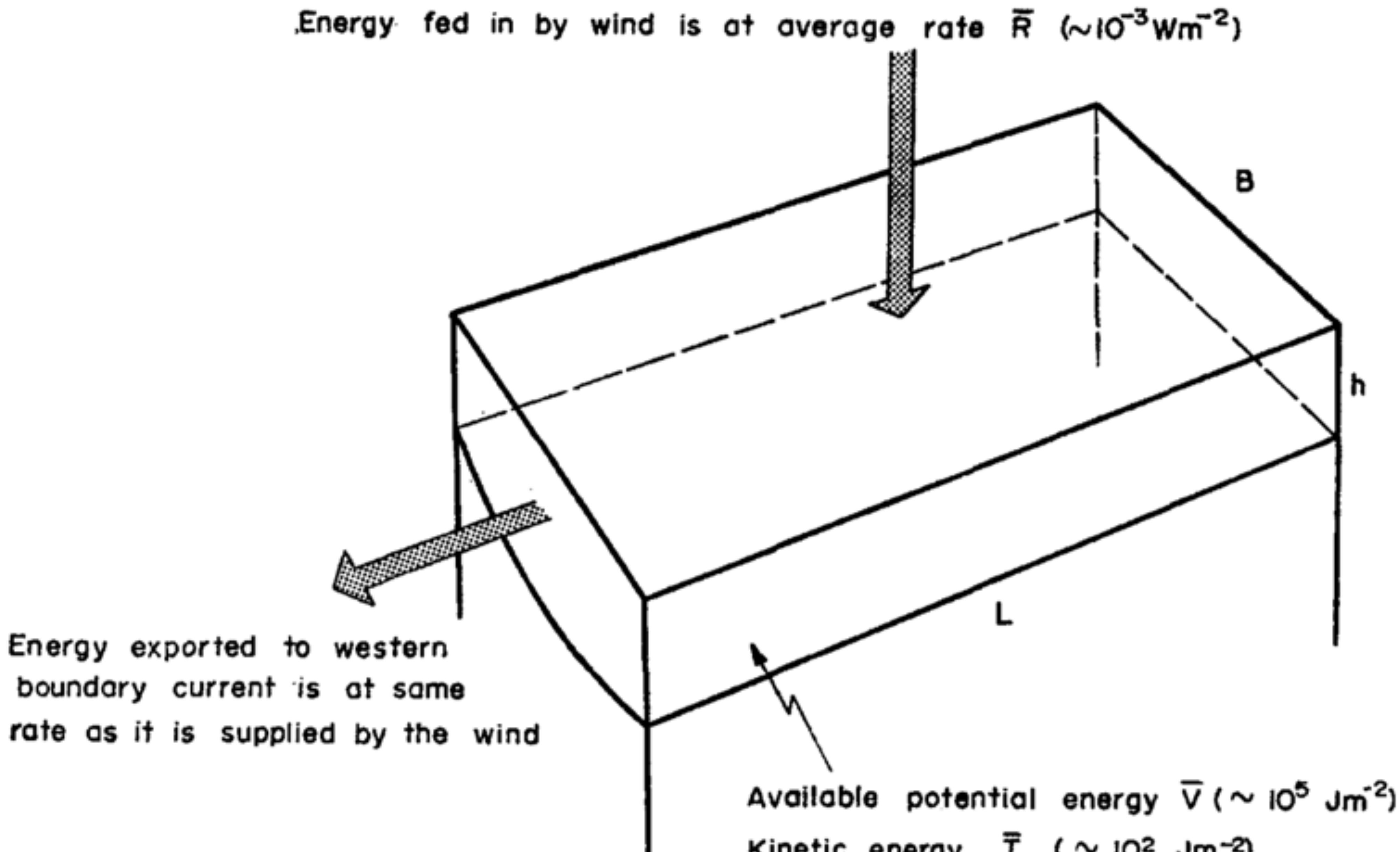
output ??? input

Rate of energy input by wind

$$\overline{R} = \rho_1 g' \overline{h w_{Ek}}$$

$$\frac{\overline{APE}}{\overline{R}} = \frac{L}{3\beta a^2} = 3\,\mathrm{yrs}$$

Energy budget



a 1m high sea state contains $10^4\,\mathrm{J/m^2}$

Eddy strength

If all the APE were converted into eddy energy,

$$\overline{E_{eddy}} = \overline{\mathrm{APE}}$$

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And for eddies larger than the deformation radius,

$$\overline{\mathrm{APE}_{eddy}} = \overline{\mathrm{APE}}$$

$$\frac{\overline{\mathrm{APE}_{eddy}}}{\overline{\mathrm{KE}_{eddy}}} \sim \frac{k^{-2}}{a^2}$$

$$\overline{\mathrm{KE}_{eddy}} = (ka)^2\overline{\mathrm{APE}} = (kB)^2\overline{\mathrm{KE}}$$

because the eddies are smaller than the gyre scale, the eddy kinetic energy can be much larger than the mean kinetic energy.

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because the eddies are smaller than the gyre scale, the eddy kinetic energy can be much larger than the mean kinetic energy.

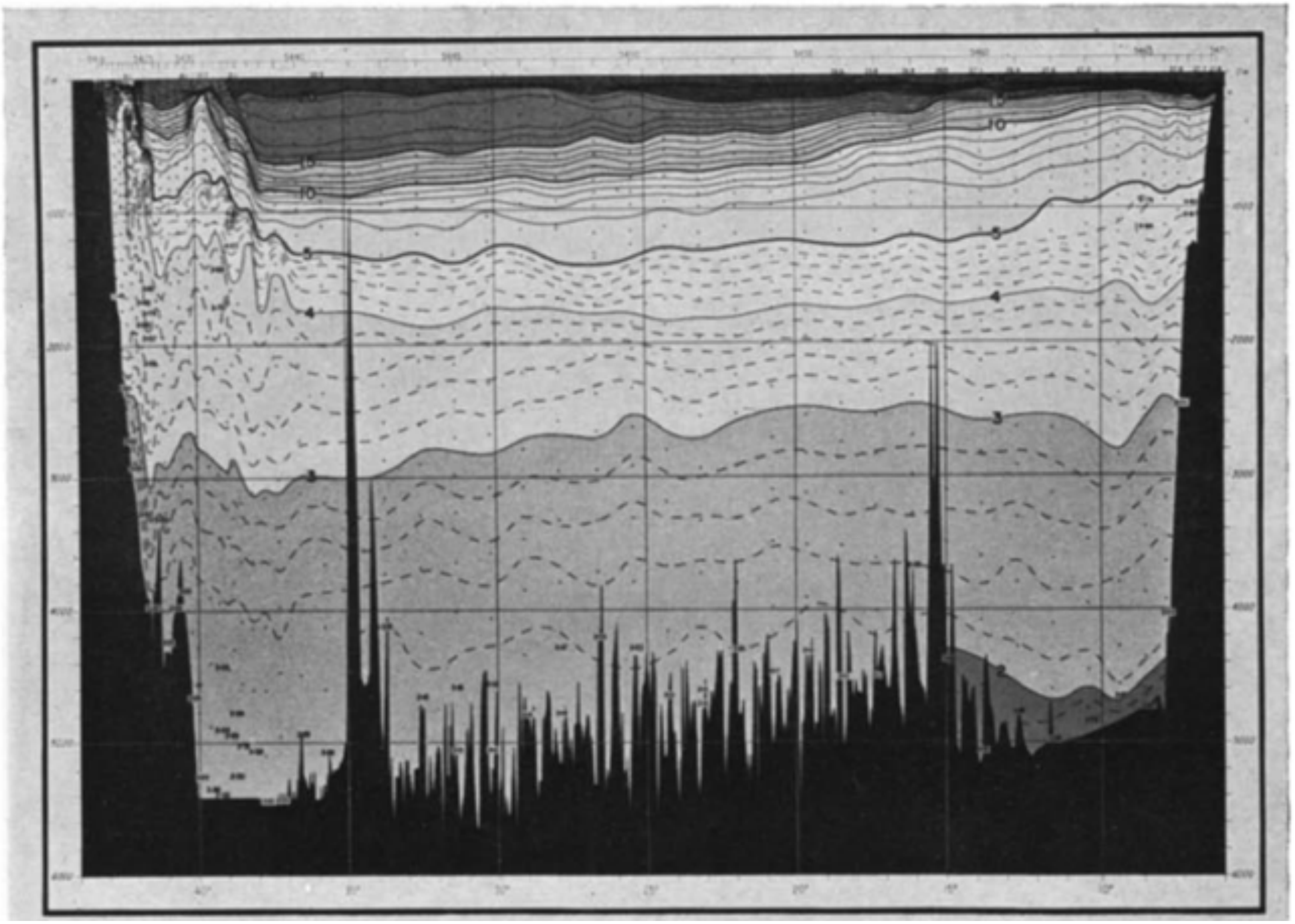
$$\overline{\mathrm{KE}_{eddy}} = (ka)^2\overline{\mathrm{APE}} \approx \overline{\mathrm{APE}}$$

$$10^5\,\mathrm{J/m^2} = \frac{1}{2}mv^2$$

→ assume barotropic eddies →

$$v = 0.2\,\mathrm{m/s}$$

How much energy is actually in eddies?



Fuglister (1960)

Meridional temperature section at 50W

15-20% of APE is in eddies

Linear stability analysis

Perturbation

$$\psi(x,y,t) = Re\{\phi(z)e^{ik(x-ct)}\}$$

Eigenvalue problem

$$(\overline{U} - c)\{[(f^2/N^2)\phi_z]_z - k^2\phi\} + Q_y\phi = 0$$

$$Q_y = \beta - [(f^2/N^2)\overline{U}_z]_z$$

Boundary conditions

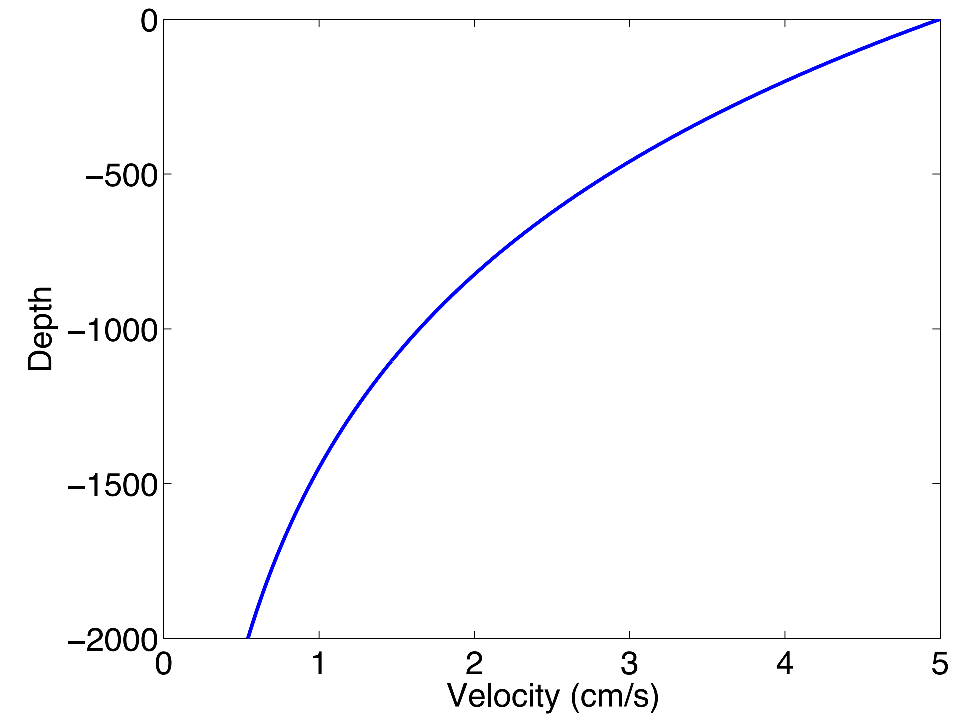
$$\frac{\phi_z}{\phi} = \frac{\overline{U}_z}{(\overline{U} - c)} \qquad z = 0$$

$$\frac{\phi_z}{\phi} = \frac{\overline{U}_z + N^2 H_y/f}{(\overline{U} - c)} \qquad z = -H$$

Necessary conditions for instability: any of

- Q_y changes sign
- Sign of Q_y is opposite to sign of $\overline{U}_z$ at $z = 0$
- Sign of Q_y is same as sign of $\overline{U}_z + \frac{N^2 H_y}{f}$ at $z = -H$

Linear stability analysis



$$(\overline{U}-c)\{[(f^2/N^2)\phi_z]_z - k^2\phi\} + Q_y\phi = 0$$

$$Q_y = \beta - [(f^2/N^2)\overline{U}_z]_z$$

Profile 1

$$\overline{U}(z) = U_0 e^{z/d} \quad \rightarrow \quad \overline{U}_z = \frac{U_0}{d}e^{z/d}$$

Assume $\frac{N^2}{d^2} = 10^4 e^{z/d}$

$\rightarrow Q_y = \beta$

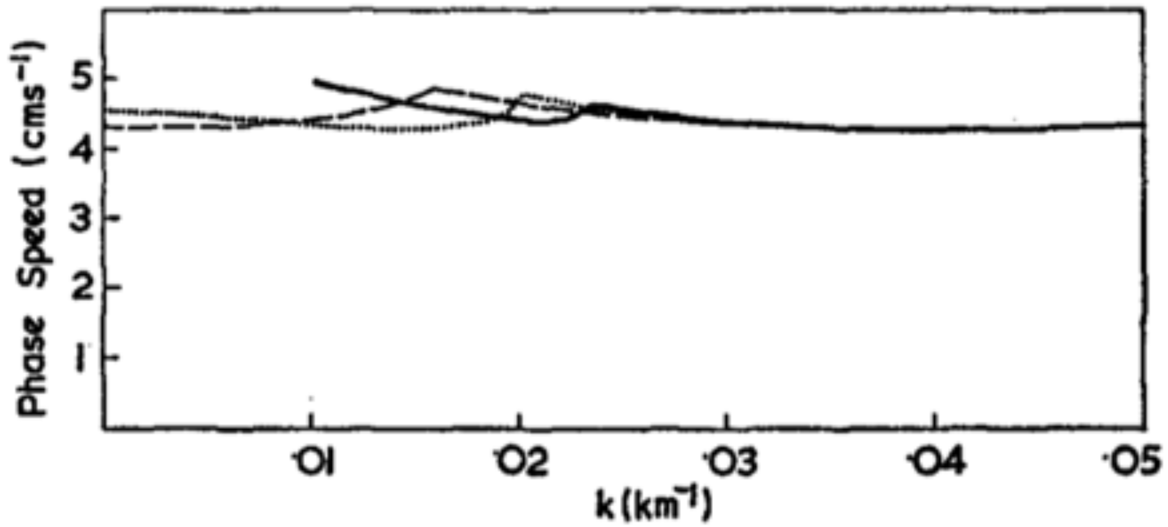
Necessary conditions for instability: any of

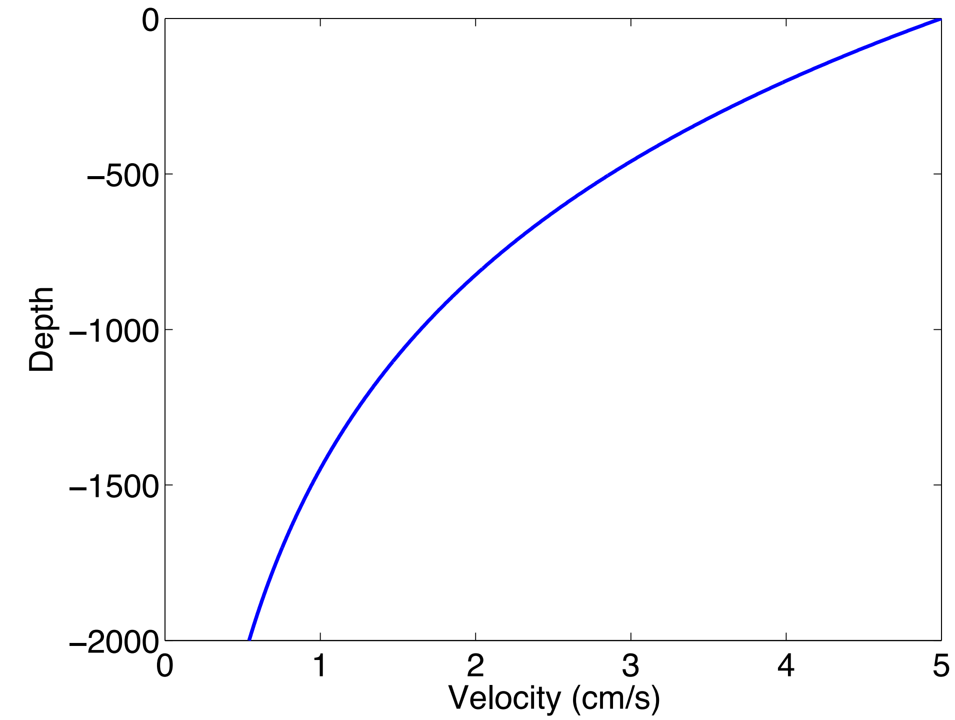
- ~~Q_y changes sign~~
- Sign of Q_y is opposite to sign of $\overline{U}_z$ at $z=0$
- ~~Sign of Q_y is same as sign of $\overline{U}_z + \frac{N^2H_y}{f}$ at $z=-H$~~ (weak)

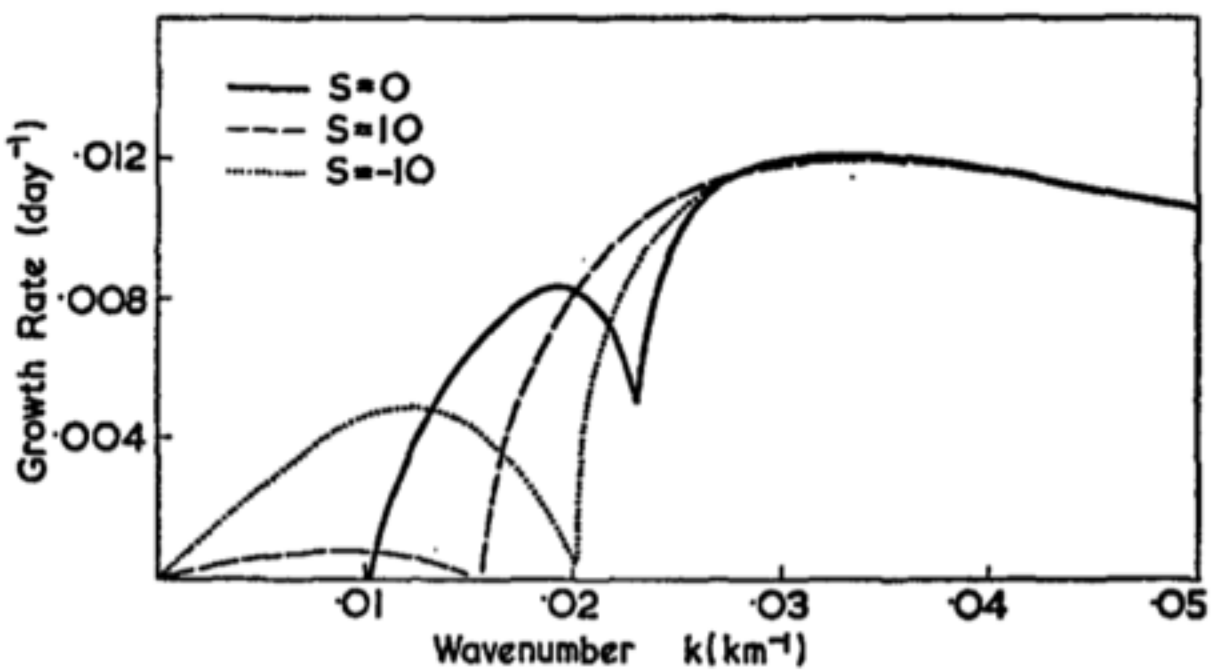
For instability $U_0 < 0$

Westward current: isopycnals slope upwards towards the equator

How is growth rate affected by the bottom slope?





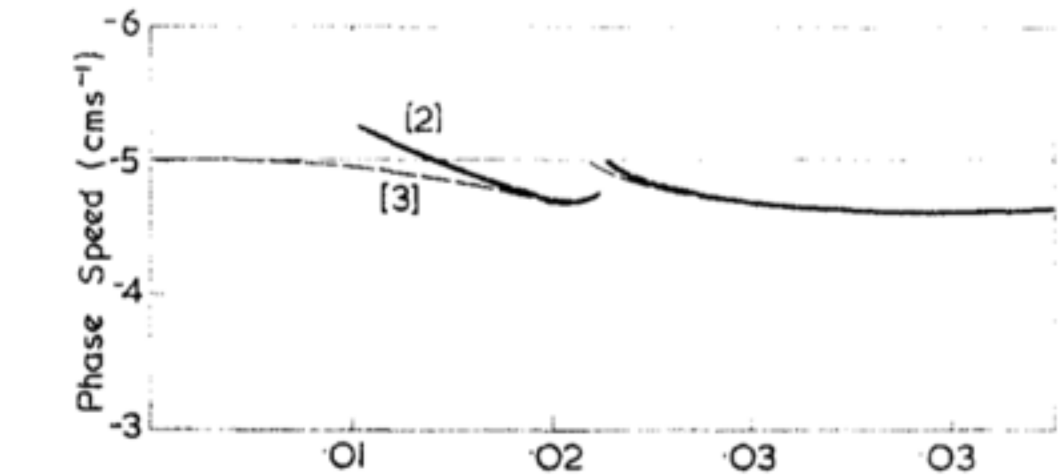


$s = H_y$ is negative when the bottom slopes in the same direction as the isopycnals.

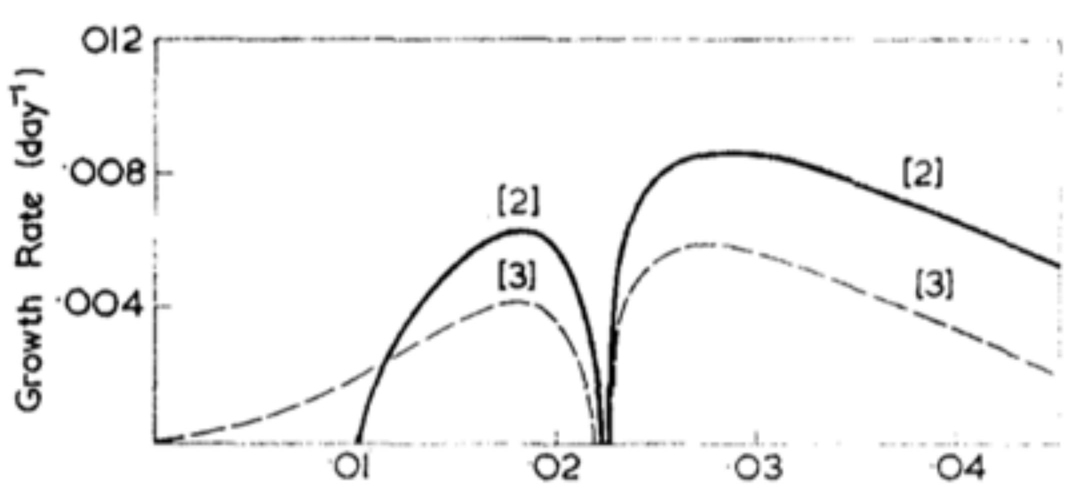
Most favorable conditions for instability when the bottom slopes upwards towards the equator

- Sign of Q_y is same as sign of $\overline{U}_z + \frac{N^2 H_y}{f}$ at $z = -H$

How is growth rate affected by the velocity profile?



$k(\mathrm{km}^{-1})$



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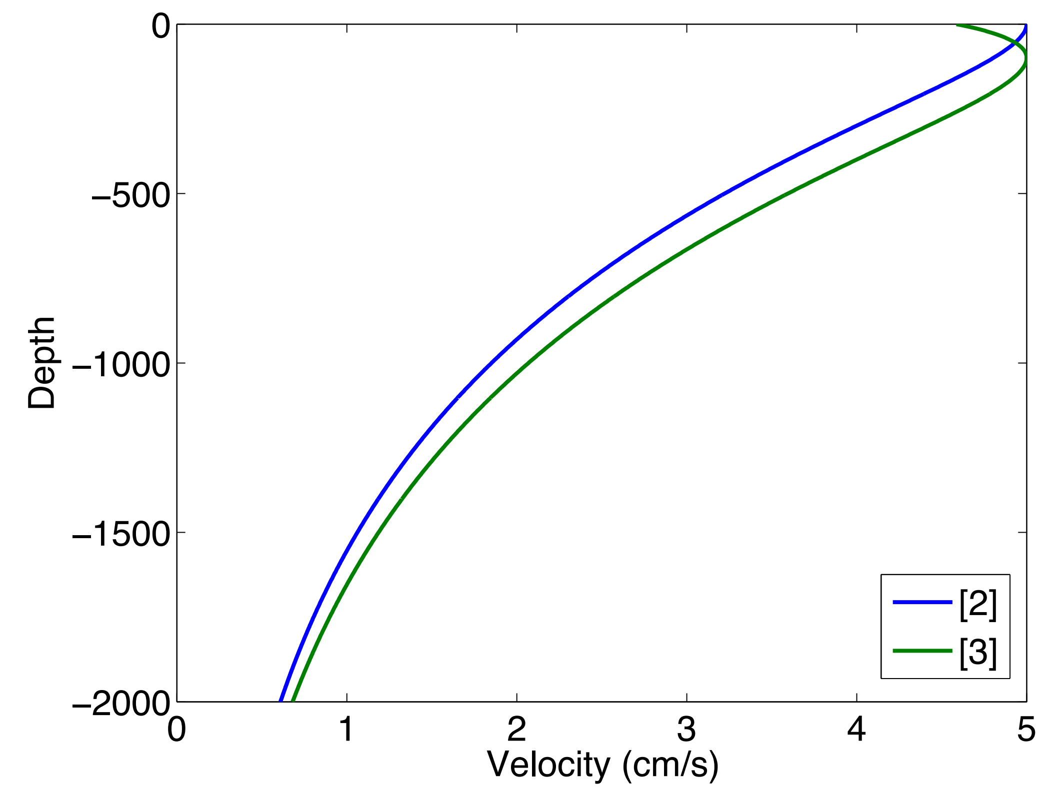
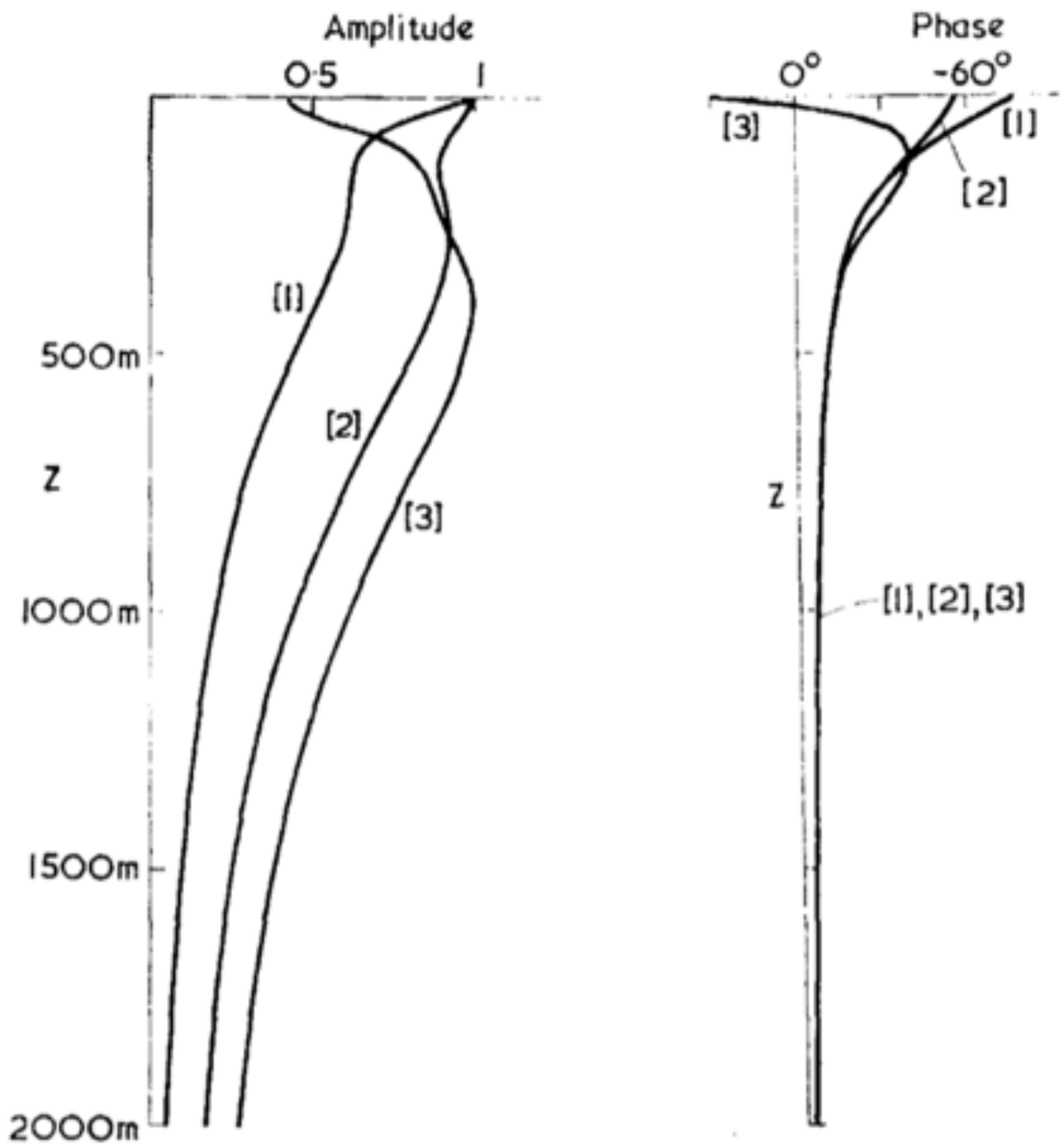


- Sign of Q_y is opposite to sign of $\overline{U}_z$ at $z = 0$

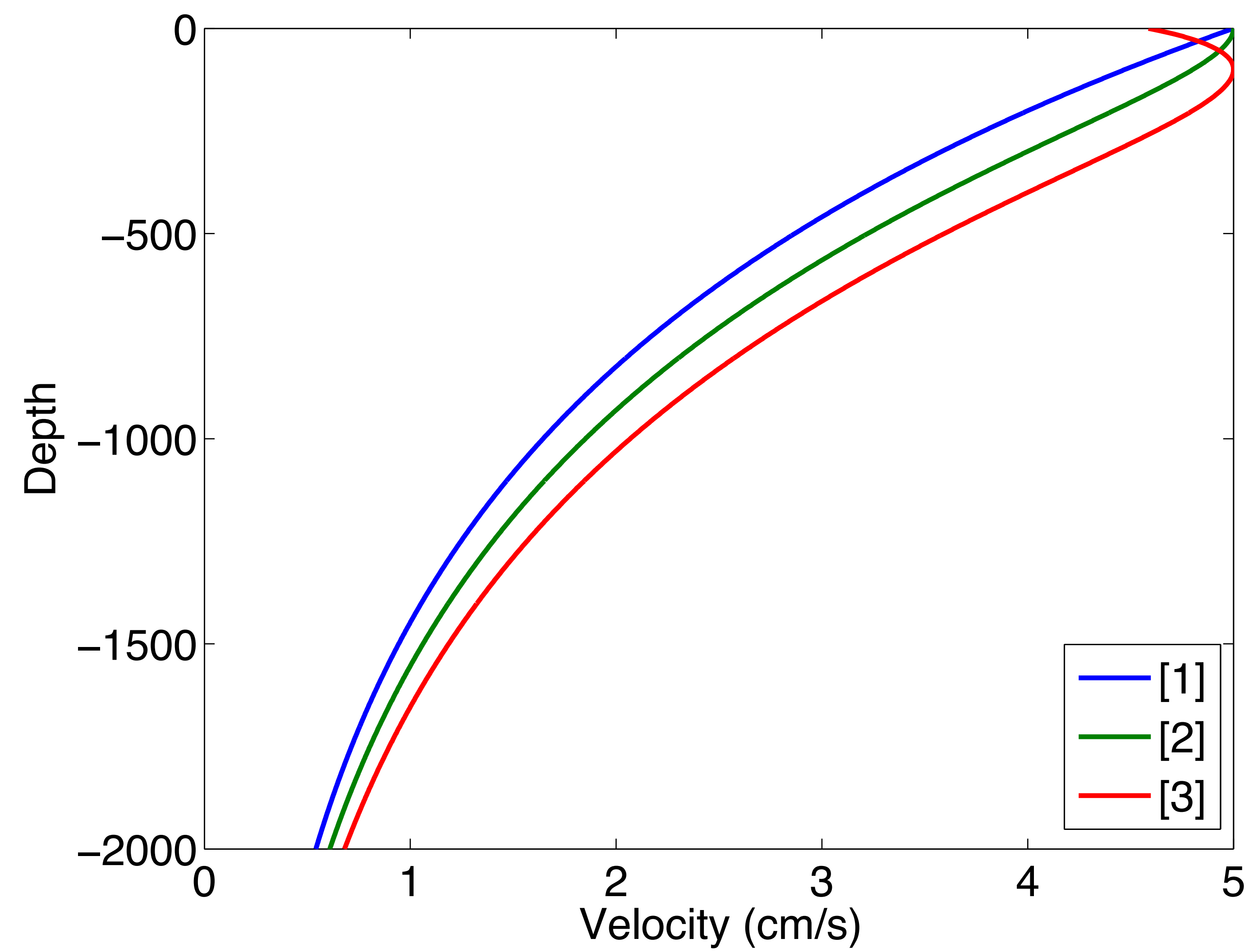
Reducing the slope of the isopycnals near the surface reduces the growth rate and the reversal of isopycnals near the surface reduces the growth rate still further.

Therefore, seasonal changes can have a large effect on the growth of disturbances.

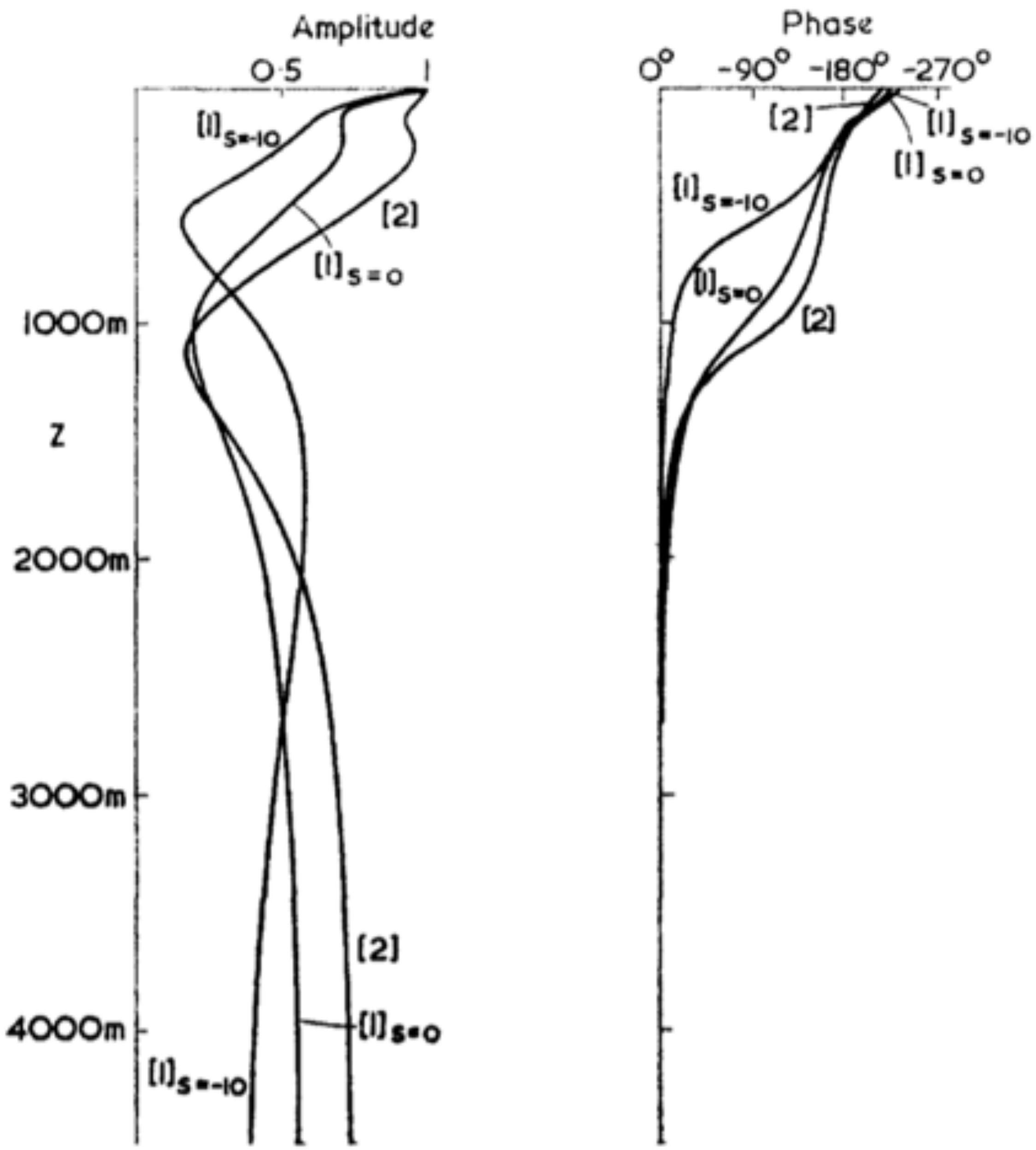
Most rapidly changing disturbances



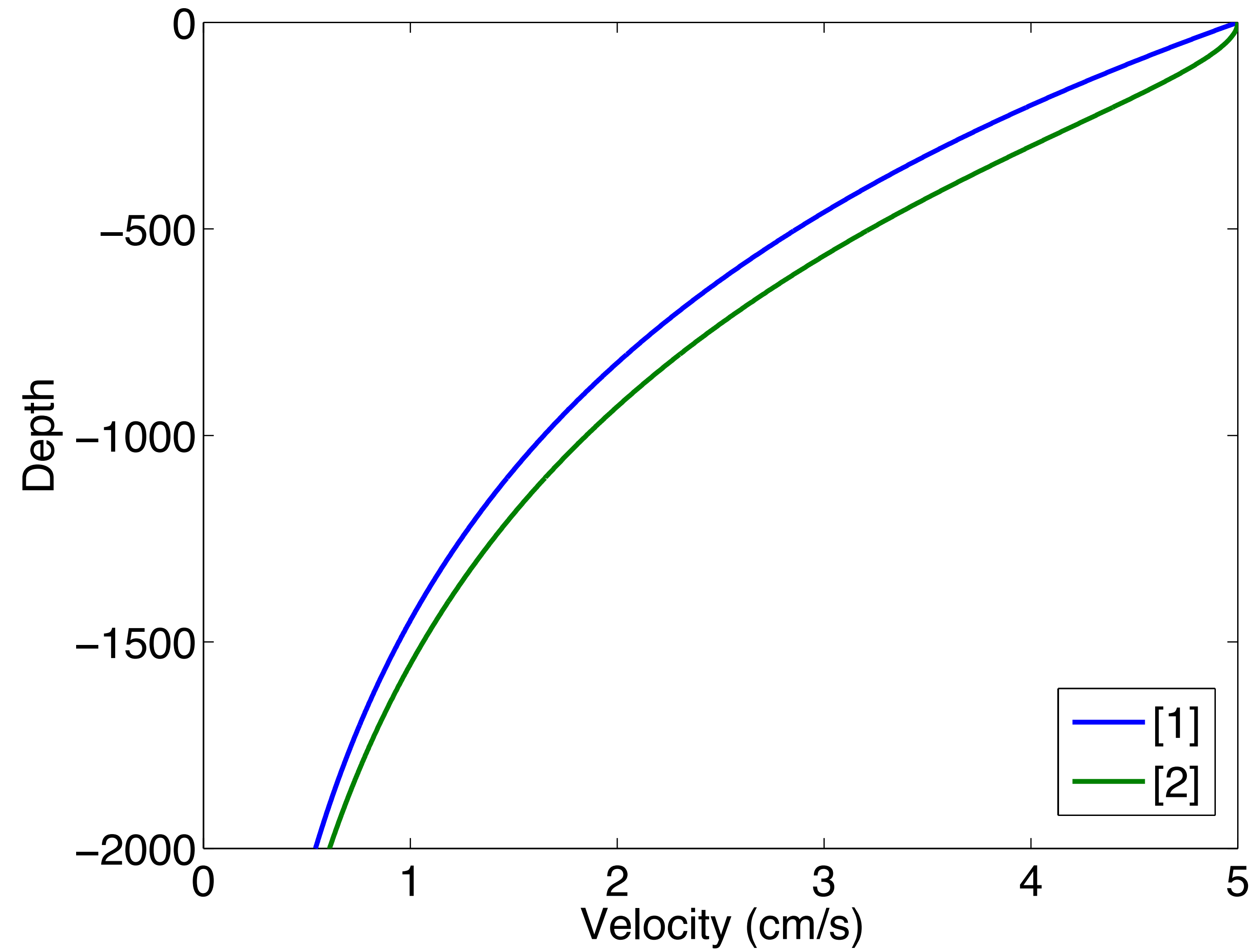
Perturbation velocity



Second most rapidly changing disturbances

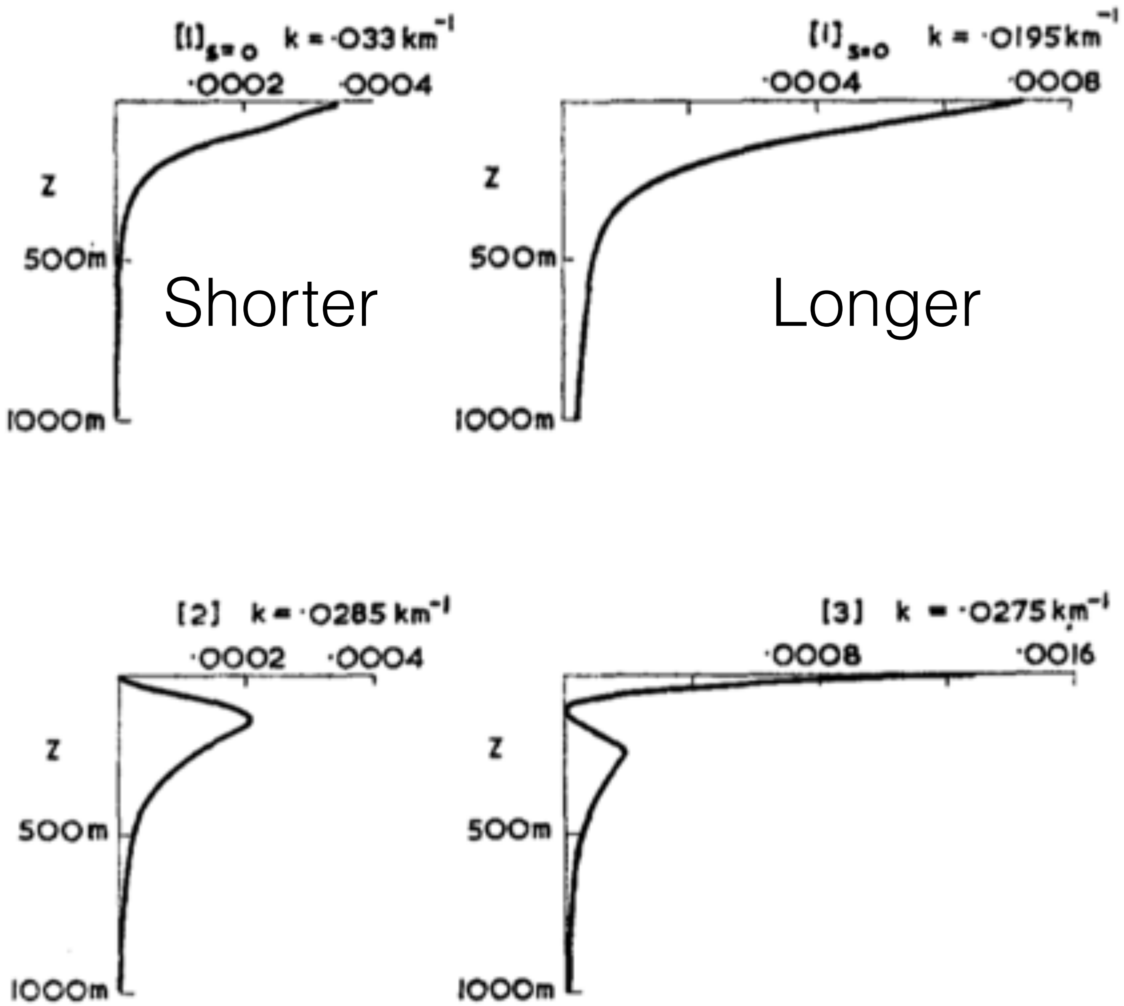


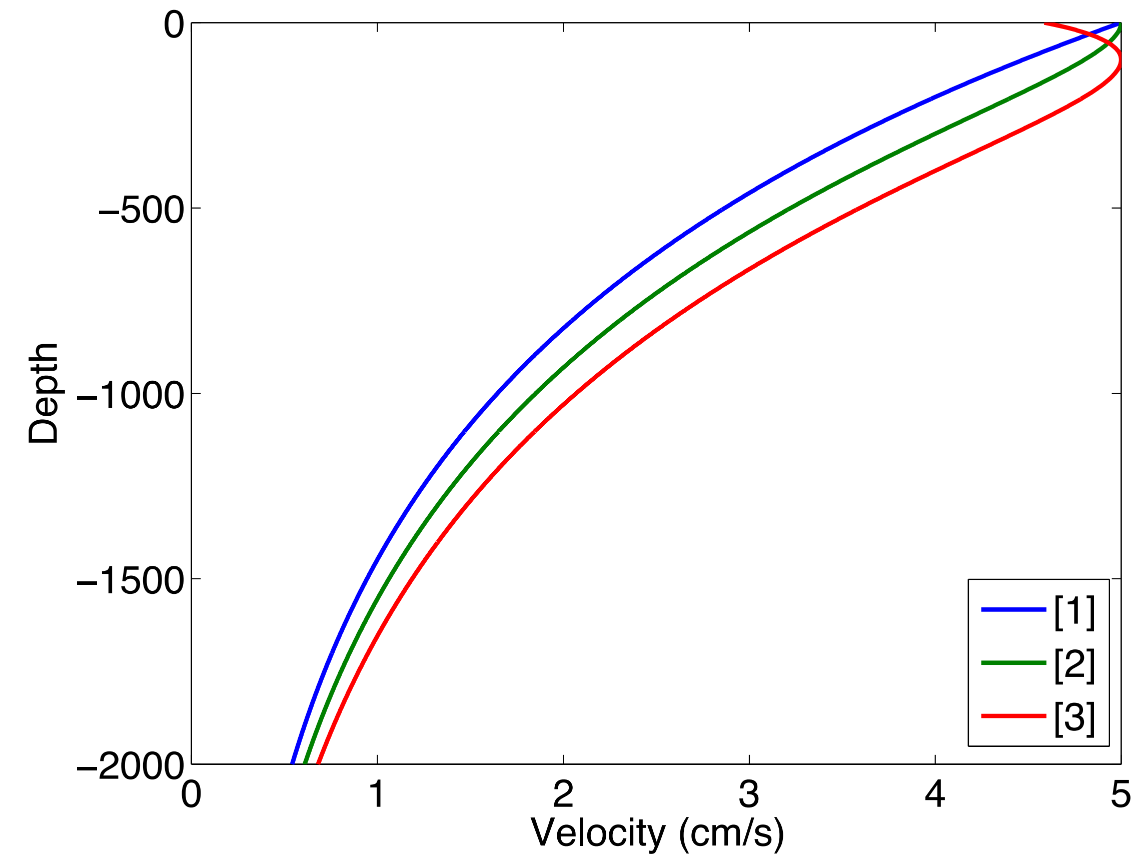
Perturbation velocity



(Similar to the first baroclinic mode)

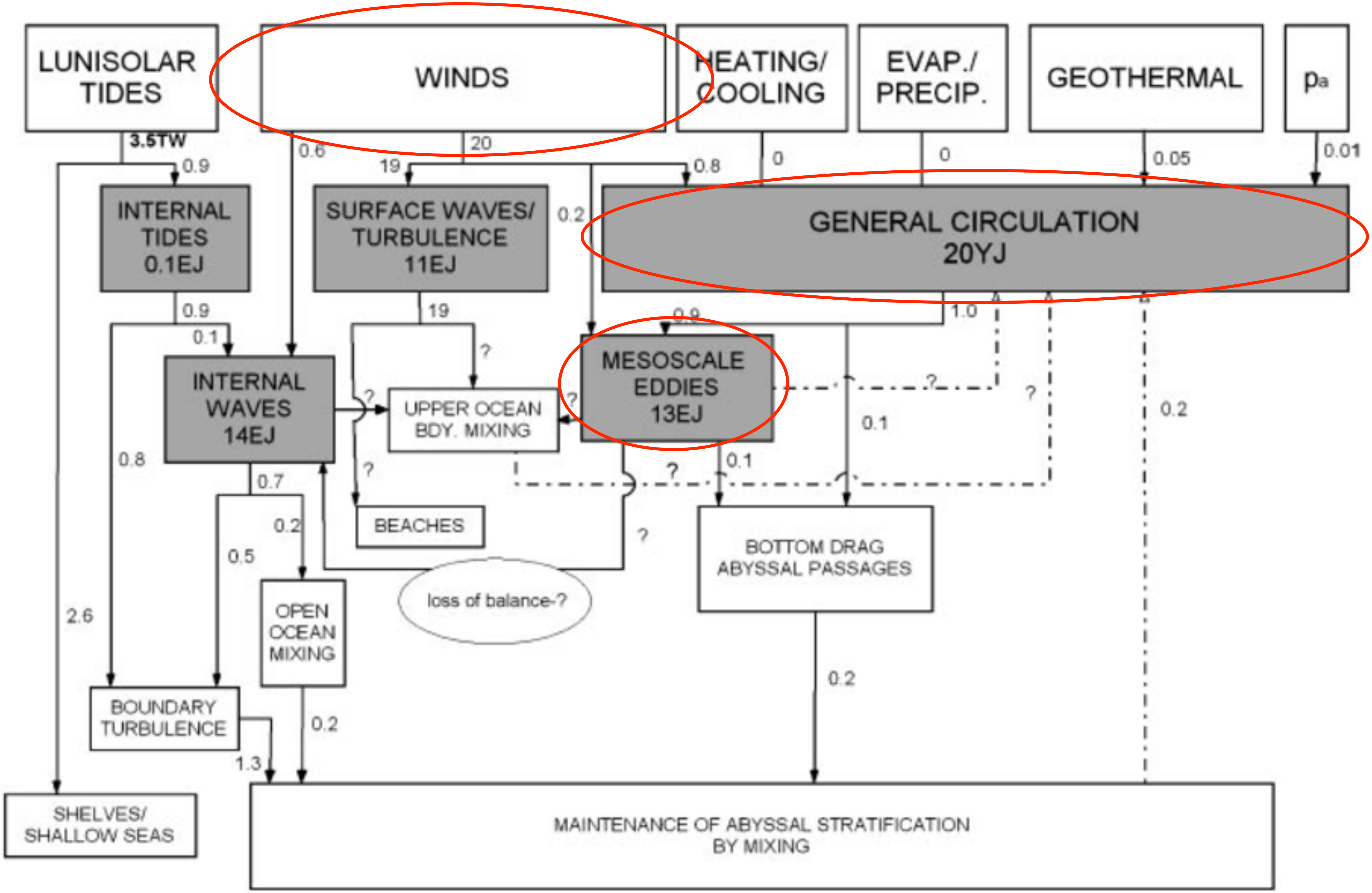
Rate of Conversion of APE to eddy energy





- Nearly all energy transfer takes place in the top 400m.
- Large eddies transfer 2-3 times as much energy as small eddies.
- Large eddies dominate at depth, small eddies at the surface.
- Eddies can remove APE from the circulation as fast as it is supplied by the wind.

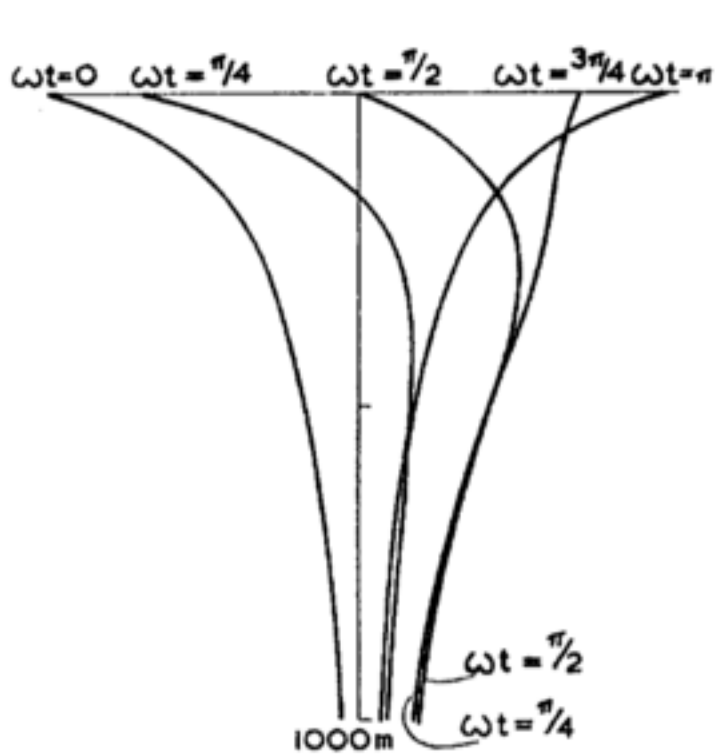
Energy in the ocean



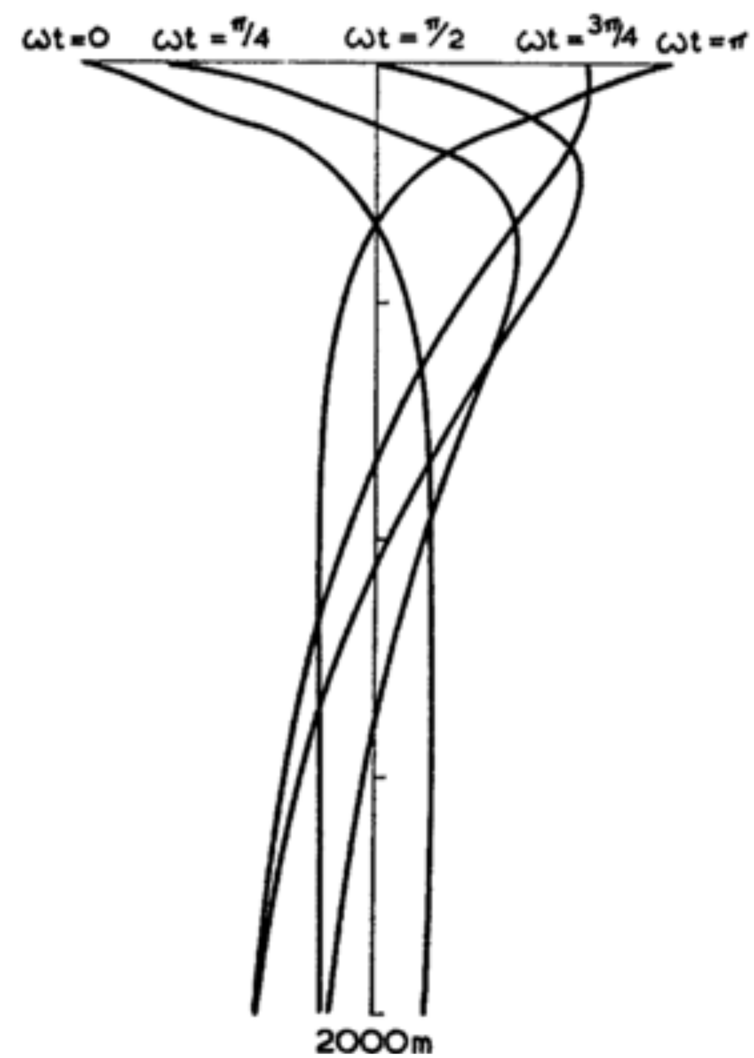
Wunsch & Ferrari (2004)

How do we identify baroclinic instability?

- Phase increases downwards in the top 400m.
- Phase velocity is upwards



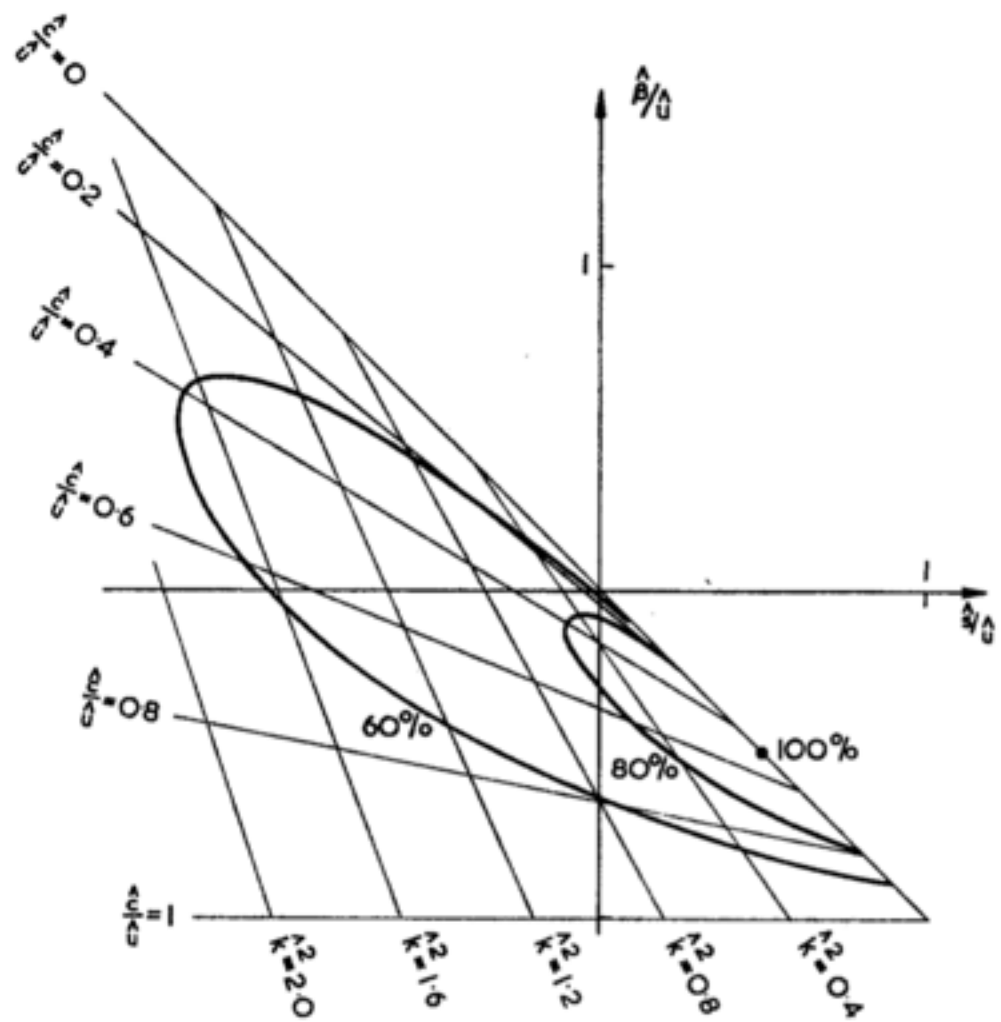
Most rapidly growing



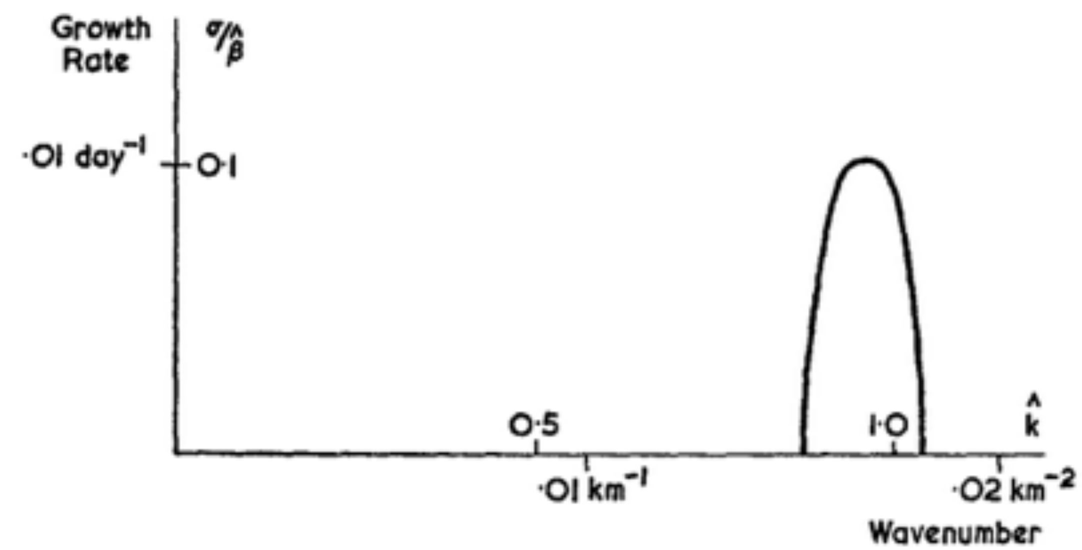
Second most rapidly growing

Failures of the two-layer model

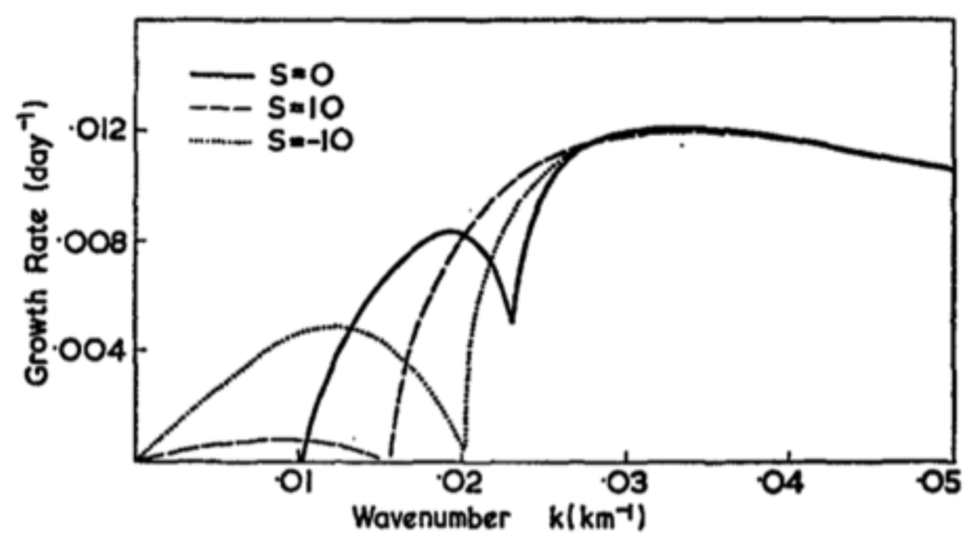
- Only Phillips-type instability is possible
- Needs tuning to give reasonable answers
- Range of wave numbers is too small



Phase speed c, wavenumber k and growth rate σ



Growth rate for 2-layer model



Growth rate for continuous model

Conclusions

- Most of the energy of the mean flow is in the APE $\frac{\overline{APE}}{\overline{KE}} \approx 1000$
- Energy from eddies comes from the large-scale mean wind field, via the large-scale mean circulation
- Smaller, faster eddies can be found in the surface field, and larger slower eddies are found at depth.
- Eddies can remove APE from the circulation as fast as it is supplied by the wind.

$$\overline{\mbox{KE}}=\frac{1}{2}\rho_1 \overline{h(u^2+v^2)}=\frac{1}{2}\rho_1 g' \overline{h(h_x^2+h_y^2)/f^2}$$

$$\overline{\mbox{KE}}\sim \frac{\rho_1 g'^2 \overline{h}\, \overline{h'^2}}{B^2f^2}$$

$$\overline{\mbox{APE}}=\frac{1}{2}\rho_1 g'(\overline{h^2}-(\overline{h})^2)$$

$$\overline{\mbox{APE}}\sim\rho_1 g'^2 \overline{h'^2}$$

$$\frac{\overline{\mbox{APE}}}{\overline{\mbox{KE}}} \sim \frac{B^2}{a^2}\approx \left(\frac{1000 \mbox{km}}{30\mbox{km}}\right)^2 \approx 1000$$

$$a=\frac{g'h^{\frac{1}{2}}}{f}\approx 30 \mbox{km}$$

$$g \eta = F_1(z)+g'h$$

$$p=\rho_1 g (\eta+h)-\rho_2 g(h+z)$$

$$(hu)_x+(hv)_y+w_{Ek}=0$$

$$-\rho_1 g' (uh^2)_x - \rho_1 g' (vh^2)_y-\rho_1 g' h w_{Ek}=0$$

$$-\rho_1 g' \left[\int uh^2 \, \mbox{d}y \right]_{x=0} - \rho_1 g' \int \int h w_{Ek} \, \mbox{d}x \mbox{d}y=0$$

$$\overline{R}=\rho_1 g' \overline{h w_{Ek}}$$

$$\frac{\overline{APE}}{\overline{R}}=\frac{L}{3 \beta a^2}=3 \, \mbox{yrs}$$

$$\overline{E_{eddy}}=\overline{\mbox{APE}}$$

$$\frac{\overline{\mbox{APE}_{eddy}}}{\overline{\mbox{KE}_{eddy}}} \sim \frac{k^{-2}}{a^2}$$

$$\overline{\mbox{KE}_{eddy}}=(k a)^2 \overline{\mbox{APE}}=(k B)^2 \overline{\mbox{KE}}$$

$$\psi(x,y,t)=Re\{ \phi(z) e^{ik(x-ct)}\}$$

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$$Q_y=\beta-[(f^2/N^2) \overline{U}_z]_z$$

$$\frac{\phi_z}{\phi}=\frac{\overline{U}_z}{(\overline{U}-c)}$$

$$\frac{\phi_z}{\phi}=\frac{\overline{U}_z+N^2H_y/f}{(\overline{U}-c)}$$

$$\mbox{Depth Scale}=\frac{f^2 \overline{U}_z}{N^2 \beta}$$

$$E=\frac{1}{2} \left(\rho_0 \left(\psi_x^2+\frac{f^2 \psi_z^2}{N^2}\right) \right)$$

$$\overline{\mbox{APE}} \sim \frac{\rho_0 f^2 \psi^2}{N^2H^2}\sim \frac{\psi^2 \rho_0}{a^2}$$