Mixed Layer Instabilities and Restratification

Boccaletti, G., Ferrari, R., and Fox-Kemper, B.
2006, J. Phys. Oceanogr., ***37****, 2228-2250*

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Abstract

The main points that the authors aim to prove are:

- Mixed layer instabilities (or MLIs) are a leading-order process in the oceanic mixed layer (ML) temperature, salinity, and momentum budgets
- These MLIs manifest themselves as baroclinic instabilities that develop on the submesoscale, and act to continuously restratify the surface ML
- MLIs can occur at a fast enough timescale to restratify between mixing events
- The release of PE and subsequent restratification is strong enough to compete with turbulent processes that work to mix the ML

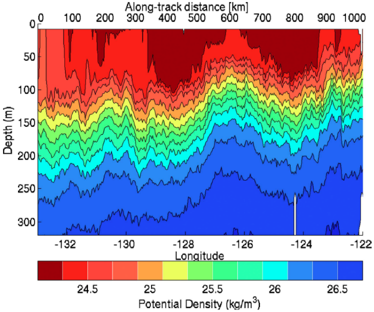
Imagine a storm...

Jon Nash, OSU

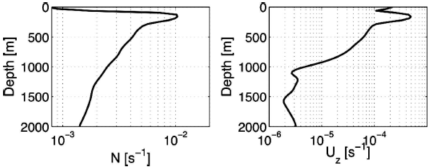
- Storm mixes the first 100 m of the ocean surface over a patch of a few hundred km^2
- Result is a homogenized layer with lateral variation in T and S
- Adjustment process
  1. Slumping of nearly vertical isopycnals via gravity
  2. Rotation modifies/slows down the slumping process, leading to geostrophic adjustment
  3. System is unstable to submesoscale eddies, and baroclinic instabilities set in
- System is restratified $\rightarrow$ another storm hits $\rightarrow$ repeat

Fundamental characteristics

CTD data from SeaSoar section - subtropical N. Pacific Ocean

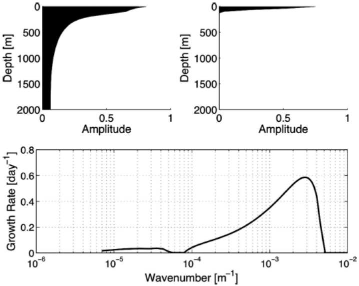


Background states used in QG stability analysis, $N = \sqrt{-g\rho_z/\rho_0}$ & $U_z = g\rho_y/f\rho_0$



- Horizontally inhomogeneous ML in upper 100 m
- Stratification & shear are weak in ML, increase suddenly across ML base, then decrease exponentially through thermocline
- In order to study whether this stratification can lead to baroclinic instability, a QG stability analysis is performed on background state

QG STABILITY ANALYSIS RESULTS



Two classes of instabilities:

1. Deep mesoscale instabilities: Scales $>$ 20 km & near 1 mo
   - Vertical structure penetrates to ocean bottom
   - Source of oceanic mesoscale eddy field
2. Shallow MLIs:
   - 200 m - 20 km, & growth scales $\sim$ 1 day
   - Trapped in surface ML
   - Energize by slumping ML density fronts

TAKE-AWAY: QG analysis shows that the mixed layer can host MLIs, which are smaller and faster than mesoscale instabilities

Geostrophy vs. ageostrophy

Consider baroclinic instability characterized by U and L, associated with disturbance developing along the front. In terms of Rossby number, $\mathrm{Ro} = U/fL$

$$L \sim \frac{U}{f\mathrm{Ro}} \quad \text{and} \quad T \sim \frac{1}{f\mathrm{Ro}}$$

Baroclinic instabilities grow near local Rossby radius of deformation,

$$\frac{NH}{fL} \sim 1$$

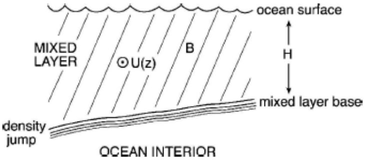
where H is the vertical scale of mode under consideration
Rewrite in terms of Ro and Ri:

$$\frac{N^2H^2}{f^2L^2} = \mathrm{Ro}^2\mathrm{Ri} \sim 1$$

where $\mathrm{Ri} = N^2H^2/U^2$ is the bulk Richardson number
This determines the scale of most unstable modes for a given stratification

- Strongly stratified: $\mathrm{Ri} \gg 1$ & $\mathrm{Ro} \ll 1 \rightarrow$ QG limit
- Weakly stratified: $\mathrm{Ri} = O(1)$ & $\mathrm{Ro} = O(1) \rightarrow$ fast ageostrophic flows

An ageostrophic model



- Assume Boussinesq adiabatic inviscid fluid
- Moving interface at $z = -H(y)$
- Jump in stratification and velocity at interface, ΔB & ΔU
- Assume ΔB at ML base is > buoyancy variations in ML

Dimensional scaling for variables:
$(x^*, y^*) = Uf^{-1}(x, y)$
$(u^*, v^*) = U(u, v)$
$(b^*, B^*, \Delta B^*) = N^2 H_0(b, B, \Delta B)$
$z^* = H_0 z$
$w^* = H_0 f w$
$p^* = N^2 H_0^2 p$
$t^* = f^{-1} t$
$H^* = H_0^{-1} H$

where $\delta = f/N$

$\rightarrow$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{Du}{Dt} = v - \mathrm{Ri}\frac{\partial P}{\partial x}$$

$$\frac{Dv}{Dt} = -u - \mathrm{Ri}\frac{\partial P}{\partial y}$$

$$\mathrm{Ri}\delta^2\left[\frac{Dw}{Dt}\right] = -\mathrm{Ri}\frac{\partial P}{\partial z} + \mathrm{Ri}b$$

$$\frac{Db}{Dt} = 0$$

An ageostrophic model cont.

Nondimensional basic state:

$$U(z) = z + 1 \quad \text{and} \quad B(y, z) = z - \frac{y}{\text{Ri}}$$

Assume perturbations of the form e^{ikx}

Linearized equations for basic state:

$$iku + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\left[\frac{\partial}{\partial t} + ikU(z)\right] u + w - v + ik\text{Ri}p = 0$$

$$\left[\frac{\partial}{\partial t} + ikU(z)\right] v + u + \text{Ri}\frac{\partial P}{\partial y} = 0$$

$$\text{Ri}\delta^2 \left[\frac{\partial}{\partial t} + ikU(z)\right] w - \text{Ri}b + \text{Ri}\frac{\partial P}{\partial z} = 0$$

$$\left[\frac{\partial}{\partial t} + ikU(z)\right] b - \frac{1}{\text{Ri}}v + w = 0$$

Boundary conditions:
$w = 0$ at $z = 0$
$w = -\partial\eta/\partial t - v(dH/dy)$ and $p = \Delta B\eta$ at $z = -1$

Stone (1970) solution

Stone (1970) solved equations with rigid lid at top and bottom boundary; looked for solutions of the form $e^{i(ly+\sigma t)}$

Found 4 types of instability:

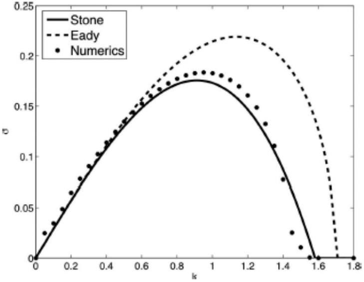
1. Baroclinic mode: most unstable wavenumber has a zonal wavenumber and vanishing meridional wavenumber $(k \to 0, l = 0)$
2. Convective mode: for $\mathrm{Ri} \leq 0$, smooths out vertical stratification
3. Symmetric mode: $(k \to 0, l \to \infty)$ grows until $\mathrm{Ri} \to 1$, then baroclinic mode takes over
4. Inertial critical layer mode: arises for ML-base slopes steeper than allowed by approximations which are used to derive BCs

For $\mathrm{Ri} < 1$, convective and symmetric instabilities develop, bring Ri to unity; for $\mathrm{Ri} > 1$, baroclinic mode takes over

Stone (1970) solution cont.

- Bulk of restratification is associated with instabilities that occur for $\text{Ri} \geq 1$
- Full ageostrophic analysis confirms that ML baroclinic mode resembles the shallow baroclinic instabilities $\rightarrow$ ML ageostrophic baroclinic instabilities are MLIs

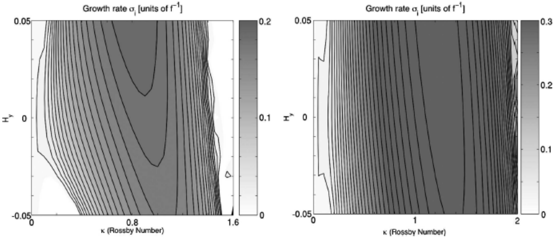
Growth rate vs. wavenumber, $\text{Ri} = 2, l = 0$



The effect of a deformable ML base

- The authors solve the eigenvalue problem with deformable bottom boundary and plot solutions for different values of H_y

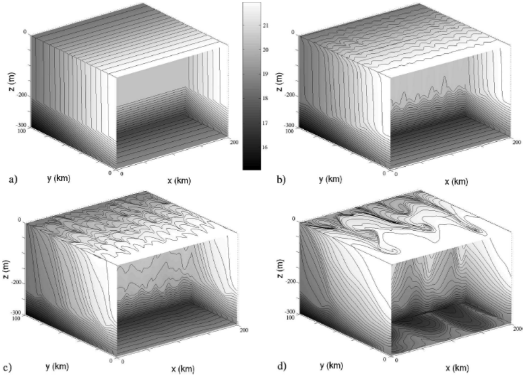
Ri = **2 & 0.5, ML base tilts -0.05-0.05**

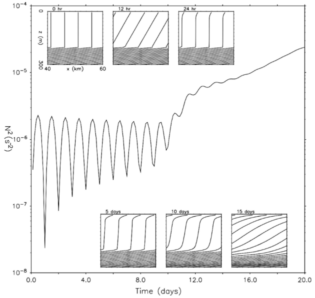


- Main effect of a tilt is to suppress instabilities at small wavenumber, and thus suppress long-wave MLIs
- TAKE-AWAY: Tilts in the ML base can explain separation of scales between mesoscale and ML instabilities

MITgcm model

- Investigate finite-amplitude development of MLIs - simulation of the adjustment of an ML front in a reentrant channel on the f plane
- Channel is 100 km $\times$ 200 km $\times$ 300 m; ML is 200 m deep
- Horizontal temperature gradient of 2.5°C across 50 km





- Vertical isopycnals oscillate around geostrophically adjusted state, $N^2 \approx B_y^2/f^2$
- Bulk of restratification begins after day 10, when MLIs reach finite amplitude

Potential vorticity budget

Main question now is: *How can MLIs achieve restratification?*

Full Ertel vorticity budget is given by

$$P = \omega_{\mathbf{a}} \cdot \nabla b$$

where $\omega_a = f\hat{\mathbf{z}} + \nabla \times u$ is absolute vorticity
Changes in PV:

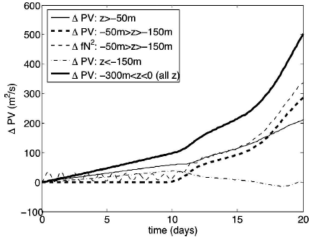
$$\frac{\partial P}{\partial t} = -\nabla \cdot \mathbf{J}, \quad \text{where} \quad \mathbf{J} = \mathbf{u}P + \nabla b \times \mathbf{F} - \omega_{\mathbf{a}} \frac{\partial B}{\partial z}$$

Advective processes like MLIs can restratify a flow by rearranging the PV in such a way that fb_z increases in the ML

Potential vorticity budget cont.

Integrate PV equation over full horizontal domain & between two vertical levels z_t and z_b, and for times 0 to t,

$$\int_{z_b}^{z_t} \left(\bar{P}|_t - \bar{P}|_{t=0}\right) \mathrm{d}z = -\int \left(\bar{J}_z|_{z=z_t} - \bar{J}_z|_{z=z_b}\right) \mathrm{d}t$$



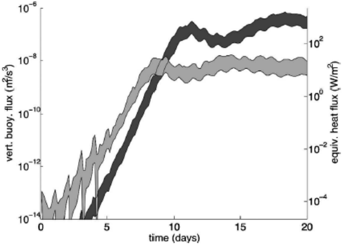
- During first 10 days, PV increases in upper & lower layers
- After day 10, fully developed eddies generate circulation in the ML, that advects high PV from surroundings → ML restratification

Potential energy budget

What controls the eddy-driven circulation?
Baroclinic eddies in a channel drive an overturning circulation in the (y,z) plane that transports PV,

$$\psi = \frac{\overline{w'b'}}{\bar{b}_y}$$

Strength of circulation $\propto$ eddy release of mean PE, $\overline{w'b'}$



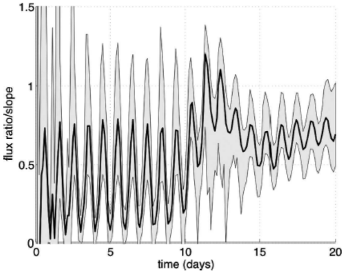
PE Extraction with fronts 5×10^{-3} °C/km (light) and 5×10^{-2} °C/km (dark) across 20 km

- PE extraction is always positive
- Release of PE by MLIs is O(1) with other ML processes for fronts $> 1 \times 10^{-3}$ °C/km
- TAKE-AWAY: Release of PE by MLIs is strong enough to compete with turbulent processes that homogenize the ML

Potential energy budget cont.

Baroclinic eddies achieve maximum release of PE by fluxing buoyancy at 1/2 the angle of the mean isopycnals (Pedlosky 1987),

$$\frac{\overline{w'b'}}{\overline{v'b'}} = -\frac{1}{2}\frac{\overline{b}_y}{\overline{b}_z}$$



$-\overline{w'b'}\,\overline{b}_z/\overline{v'b'}\,\overline{b}_y$ vs. time

- The eddy flux slope oscillates around 1/2 the isopycnal slope for most of the simulation

Potential energy budget cont.

- In all simulations, ψ tracks evolution of $\overline{w'b'}$
- ψ is not affected by diurnal forcing, because vertical mixing due to surface heating/cooling does not affect either $\overline{w'b'}$ or $\overline{b}_y$ $\rightarrow$ eddy-driven circulation is independent of frictional and diabatic processes
- Rate of restratification is affected by external forcing, because frictional/diabatic processes are the only ones that can modify the PV state on which ψ acts

Conclusion

- Lateral buoyancy gradients are created through surface fluxes or mesoscale straining
- These gradients will slump under the action of gravity
- Rotation constrains the efficiency of the slumping, because thermal wind balance is established
- Ageostrophic baroclinic instabilities (MLIs) allow for restratification by creating wavelike disturbances that upset the balance
- MLIs rapidly reach finite amplitude and tilt isopycnals from vertical to horizontal
- MLIs develop on the submesoscale and work fast enough to restratify between mixing events
- Finite-amplitude MLIs inject high PV waters into ML by driving large vertical velocities that cause entrainment