Isopycnal Mixing in Ocean Circulation Models†

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ABSTRACT

A subgrid-scale form for mesoscale eddy mixing on isopycnal surfaces is proposed for use in non-eddy-resolving ocean circulation models. The mixing is applied in isopycnal coordinates to isopycnal layer thickness, or inverse density gradient, as well as to passive scalars, temperature and salinity. The transformation of these mixing forms to physical coordinates is also presented.

1. Introduction

It is now half a century since Iselin (1939) and Montgomery (1940) suggested that mixing of material properties by eddies in the stably stratified parts of the oceans occurs mostly along surfaces of constant density or isopycnal surfaces. The most energetic component of the eddy field is on the mesoscale, and mesoscale eddies dominate the isopycnal mixing of material properties.

It is presently common practice to model mesoscale eddies and their interaction with the wind-driven general circulation with an adiabatic approximation; that is, density is assumed to be conserved following fluid parcels, and all of the dissipative processes are confined to the momentum equations. The resulting numerical solutions (e.g., Holland 1978) are self-consistent in that the density fields are spatially smooth and surfaces of constant density do not undergo overturning and breaking motions. Of course, if we could obtain a complete solution of the Navier–Stokes equations, we would expect some degree of turbulent cascade from the planetary scale, through the mesoscale, to the smaller scales where breaking and molecular dissipation will act on the density field. However, we cannot yet demonstrate that this pathway is essential in this wind-driven circulation problem, even though it is essential to achieving an equilibrium state in the presence of buoyancy fluxes through the boundaries, the so-called diabatic problem.

In a fine-resolution numerical solution for the adiabatic, wind-driven general circulation, an analysis of the density equation shows that, in a statistical steady state, the three-dimensional divergence of the flux of the mean (large-scale) density field by the mean velocity is balanced by the divergence of a mean density flux due to mesoscale eddies. In a sufficiently coarse-resolution model of this situation, mesoscale eddies are not present. So, in order to mimic a fine-resolution model, the eddy density flux divergence must be represented by a subgrid-scale parameterization in order to have a sensible resolved-scale density balance. Therefore the coarse-resolution model must have a nonconservative term in the density equation. Thus, the mean or large-scale density equation is not pointwise adiabatic, but our parameterization of eddy mixing will retain the integral properties of the density field in adiabatic flow, and so we call it quasi-adiabatic (see below). Thus, our coarse-resolution model should be thought of as an approximate model that preserves the important properties of adiabatic evolution. Even in a more complete general circulation model, with surface buoyancy forcing and diapycnal (perpendicular to isopycnal) mixing by eddies on scales smaller than mesoscale, the mesoscale eddy isopycnal density fluxes will still be an important contributor to the steady-state density balance, and so they still must be parameterized in a coarse-resolution model, together with whatever diapycnal subgrid-scale fluxes are appropriate.

Analogous considerations apply to the transport of passive tracers. In contrast to the density field, however, mean and mesoscale advection cause a vigorous cascade of tracer variance to small scales, so that its dissipation rate will become significant no matter how small the tracer diffusivity. In an adiabatic model of a stratified fluid, tracer mixing can only occur along isopycnal surfaces. So the behavior of the fine-resolution solutions that we wish to mimic in coarse-resolution

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solutions is conservation of mean tracer concentration on isopycnals and decay of all higher moments of the concentration. This, too, is part of our quasi- adiabatic parameterization proposal.

Previous proposals for isopycnal mixing in ocean models (Kirwan 1969; Solomon 1971; Redi 1982; Cox 1987) have been eddy diffusion laws for passive scalars with the flux vector rotated to a coordinate frame locally tangent to an isopycnal surface, and with different eddy diffusivities for the isopycnal and diapycnal components. McDougall (1987a,b) has extended this proposal to neutral surfaces rather than isopycnals. However, when such a law is applied to density, the isopycnal flux is zero. Thus, these proposals fail to satisfy the requirements of the mean density balance as discussed above.

We propose forms for isopycnal eddy mixing that overcome this deficiency in a way which permits qualitative correspondence between adiabatic eddy-resolving solutions and coarse-resolution solutions in a quasi-adiabatic, approximate model. The proposal is to mix isopycnal layer thickness, or inverse density gradient, along isopycnal surfaces and to mix passive scalars with an additional term compared to previous proposals. In addition, diapycnal diffusion may be included, as it must be for diabatic problems, but we will mostly neglect this aspect in our discussion here. We also note here that we will not discuss horizontal eddy motions, which contribute to horizontal Reynolds stresses. Section 2 contains the proposed forms for isopycnal mixing, which are natural in isopycnal coordinates. In section 3, these forms are transformed into physical, or height, coordinates. Finally, section 4 is a discussion that includes the generalization of our proposal to the equation of state for sea water.

2. Mixing in isopycnal coordinates

a. *Eddy-resolving models*

In isopycnal coordinates, the adiabatic density conservation and incompressible continuity equations are combined to give an equation for the thickness, $\partial h/\partial\rho$, where ρ is density and $h(x, y, \rho, t)$ is the physical height of a density surface. The equation is

$$\frac{\partial^2 h}{\partial t \partial \rho} + \nabla_\rho \cdot \left(\frac{\partial h}{\partial \rho}\mathbf{u}\right) = 0, \tag{1}$$

where $\mathbf{u}$ is the horizontal velocity vector and ∇_ρ is the horizontal gradient operator applied at constant ρ.

If the diffusion of a passive scalar, τ, is assumed to occur only along, and not across, isopycnal surfaces, then the equation for τ is

$$\frac{D\tau}{Dt} = \nabla_\rho \cdot \left(\mu \frac{\partial h}{\partial \rho}\mathbf{J}\cdot\nabla_\rho\tau\right)\Big/\frac{\partial h}{\partial \rho} \equiv R(\tau), \tag{2}$$

where μ is the tracer diffusivity. The adiabatic substantial derivative in isopycnal coordinates is given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\rho, \tag{3}$$

and

$$\mathbf{J} = \frac{1}{1 + h_x^2 + h_y^2}\begin{bmatrix} 1 + h_y^2 & -h_x h_y \\ -h_x h_y & 1 + h_x^2 \end{bmatrix}. \tag{4}$$

$\mathbf{J}$ is not the identity matrix because isopycnal coordinates are not orthogonal. In the ocean the slopes of the isopycnals are generally very small and, in the limit that horizontal density gradients are much smaller than vertical gradients, $\mathbf{J}$ reduces to the identity matrix. In this limit, Eq. (2) reduces to the simple flux form in isopycnal coordinates.

In an adiabatic eddy-resolving model, Eqs. (1) and (2) have the following important properties:

A) All domain-averaged moments of ρ are conserved, and the volume between any two isopycnals is conserved.

B) With insulating boundary conditions, the domain-average of τ is conserved between any two isopycnals, and higher moments of τ decrease in time if τ has gradients on the isopycnals.

C) Since $R(\rho)$ is zero, Eq. (2) for a passive tracer, τ, is satisfied identically by the density, ρ (see the discussion in section 4).

If an eddy-resolving, adiabatic, isopycnal model is run to a statistical steady state, then the approximate balance in Eq. (1) will be

$$\nabla_\rho \cdot \left(\frac{\partial \bar{h}}{\partial \rho}\bar{\mathbf{u}}\right) + \overline{\nabla_\rho \cdot \left(\frac{\partial h'}{\partial \rho}\mathbf{u}'\right)} \approx 0. \tag{5}$$

Here the thickness and velocity are divided into large-scale, time-mean ($\bar{\mathbf{u}}$) and eddy components ($\mathbf{u}'$), and the overbar represents an average over the eddy scales. The second term of Eq. (5) is the isopycnal mixing contribution due to eddies, and it appears as a source term in the thickness equation for the large-scale variables.

b. *Non-eddy-resolving models*

Therefore, the eddy mixing can be represented in approximate non-eddy-resolving models by the equation

$$\frac{\partial^2 h}{\partial t \partial \rho} + \nabla_\rho \cdot \left(\frac{\partial h}{\partial \rho}\mathbf{u}\right) + \nabla_\rho \cdot \mathbf{F} = 0. \tag{6}$$

Now that there is a nonconservative term in the thickness equation (6), there is a choice to be made as to whether it corresponds to a term in the adiabatic density or incompressible continuity equations. If the flow is considered strictly adiabatic, then there must be an extra term in the continuity equation and the flow is not incompressible. To us, this is an unfamiliar and conceptually uncomfortable ocean model because the

flow will have mass sources and sinks. We prefer the alternative which is to consider the flow as strictly incompressible and to have a nonconservative term in the density equation.

Thus, our non-eddy-resolving model is not adiabatic, and so particles do not preserve their density, i.e.,

$$\frac{D\rho}{Dt} = Q, \tag{7}$$

where Q is related to the nonconservative term in Eq. (6) by

$$\frac{\partial h}{\partial \rho} Q = \int^{\rho} \nabla_\rho \cdot \mathbf{F} d\rho. \tag{8}$$

From its definition and by Eq. (7), the diabatic substantial derivative in isopycnal coordinates for the non-eddy-resolving model is given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\rho + Q \frac{\partial}{\partial \rho}. \tag{9}$$

The Q term in the substantial derivative (9) may be unfamiliar. However, the coarse-resolution model is diabatic, and in isopycnal coordinates this is represented by the Q term in the substantial derivative.

Despite the fact that the non-eddy-resolving model is locally diabatic, we will ensure that the three important properties of the adiabatic, eddy-resolving model listed in section 2a are still retained by the non-eddy-resolving model. Thus, we refer to our non-eddy-resolving model as having quasi-adiabatic evolution. The first property A is assured by the boundary conditions that $\mathbf{F} \cdot \mathbf{n}$ is zero on all boundaries, where $\mathbf{n}$ is the normal horizontal vector, and Q is zero on the vertical boundaries of the domain.

We consider now the equation for passive scalars in the non-eddy-resolving model. Guided by Eq. (2), but allowing for a nonconservative term, E, we write the equation for τ as

$$\frac{D\tau}{Dt} = R(\tau) + E(\tau) \Big/ \frac{\partial h}{\partial \rho}. \tag{10}$$

In order to satisfy the τ conservation properties B from section 2a, it is easy to show that the source term E must be of the form

$$E(\tau) = \frac{\partial}{\partial \rho} \left(\frac{\partial h}{\partial \rho} Q\tau \right) + \nabla_\rho \cdot \mathbf{G}(\tau). \tag{11}$$

To satisfy C from section 2a, that ρ satisfies Eq. (10) identically, requires that

$$\frac{\partial h}{\partial \rho} Q = E(\rho). \tag{12}$$

Using equations (8) and (11), Eq. (12) can be written in the form

$$\nabla_\rho \cdot [\rho \mathbf{F} + \mathbf{G}(\rho)] = 0. \tag{13}$$

Using the simplest solution of Eq. (13) results in the following form for the passive scalar equation (10)

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\rho \right) \tau + \mathbf{F} \cdot \nabla_\rho \tau \Big/ \frac{\partial h}{\partial \rho} = R(\tau). \tag{14}$$

Comparing Eqs. (2) and (14) shows that there is a single extra term in the passive scalar equation in the non-eddy-resolving model, which has the form of an additional horizontal advection of τ by the isopycnal thickness flux. In addition, the value of μ used in Eq. (14) will be much larger than in the eddy-resolving model equation (2).

c. A simple choice for $\mathbf{F}$

A simple choice for $\mathbf{F}$ is

$$\mathbf{F} = -\frac{\partial}{\partial \rho} (\kappa \nabla_\rho h), \tag{15}$$

with $\mathbf{F}$ zero on the boundaries. The thickness diffusivity, κ, can be spatially varying, but, if it is a constant, Eq. (6) has the familiar form of Laplacian mixing acting upon the equation variable. The mixing of isopycnal thickness, or the inverse of the density gradient, seems to us the simplest, nontrivial mixing formulation in isopycnal coordinates. The other reason why we like the choice (15) is that it makes Q, given by Eq. (8), a local function. In fact, Q is given by

$$\frac{\partial h}{\partial \rho} Q = -\nabla_\rho \cdot (\kappa \nabla_\rho h), \tag{16}$$

with Q zero on the vertical boundaries of the domain.

The choice for $\mathbf{F}$ in (15) has certain implications for the structure of the resulting flow field that are most easily illustrated in a simplified geometry. The Appendix contains the analysis for flow in a zonally uniform channel in the geostrophic limit of small Rossby number. It shows that, when κ is positive, there is downgradient vertical diffusion of mean (large-scale) momentum and there is a conversion of mean potential energy to eddy (subgrid-scale) potential energy. The former process is often referred to as isopycnal or interfacial form drag. The latter exchange simulates the potential energy conversion due to baroclinic instability, a process which dominates energy conversion in broad, midlatitude zonal currents in eddy-resolving ocean circulation models, see McWilliams and Chow (1981).

In ocean models it is desirable to have positive vertical momentum diffusion and potential energy conversion from mean to eddy in the global domain average, but this does not always occur locally. In eddy-resolving solutions the largest discrepancy usually occurs in western boundary currents where momentum diffusion and potential energy conversion are the opposite of those described above, see McWilliams et al. (1989). Thus, it remains an issue for implementation

whether inclusion of these effects by making κ a function of space and time (e.g., with either reduction, or sign reversal, of κ in western boundary currents) is important or needed.

3. Transformation to physical coordinates

a. General equations

Transforming Eqs. (7) and (8) to physical coordinates gives the density equation with a nonconservative term in the form

$$\frac{D\rho}{Dt} = Q, \tag{17}$$

where D/Dt is the familiar substantial derivative in physical coordinates, and Q is given by

$$\frac{\partial}{\partial z}(Q/\rho_z) = \rho_z \nabla_z \cdot \mathbf{F} - \nabla_z \rho \cdot \frac{\partial \mathbf{F}}{\partial z}, \tag{18}$$

and ∇_z is the horizontal gradient operator applied at constant z. The passive scalar equation (14) for τ transforms into the advective form

$$\frac{D\tau}{Dt} + \rho_z \mathbf{F} \cdot \nabla_z \tau - [\mathbf{F} \cdot \nabla_z \rho + Q/\rho_z] \frac{\partial \tau}{\partial z} = R(\tau). \tag{19}$$

Using Eq. (18), Eq. (19) can also be written in flux form. The transformation of R from Eq. (2) takes the form

$$R(\tau) = \nabla \cdot (\mu \mathbf{K} \cdot \nabla \tau), \tag{20}$$

where ∇ is the 3-D gradient operator applied at constant z, and $\mathbf{K}$ has the following form, see Redi (1982):

$$\mathbf{K} = \frac{1}{\rho_x^2 + \rho_y^2 + \rho_z^2} \times \begin{bmatrix} \rho_y^2 + \rho_z^2 & -\rho_x\rho_y & -\rho_x\rho_z \\ -\rho_x\rho_y & \rho_x^2 + \rho_z^2 & -\rho_y\rho_z \\ -\rho_x\rho_z & -\rho_y\rho_z & \rho_x^2 + \rho_y^2 \end{bmatrix}. \tag{21}$$

In the limit that horizontal density gradients are much smaller than vertical gradients, there is an approximation to $\mathbf{K}$ which preserves the isopycnal form of mixing; i.e., $R(\rho)$ is still identically zero. It is the exact transformation of approximating $\mathbf{J}$ in (4) by the identity matrix, and is given by

$$\mathbf{K}' = \begin{bmatrix} 1 & 0 & -\rho_x/\rho_z \\ 0 & 1 & -\rho_y/\rho_z \\ -\rho_x/\rho_z & -\rho_y/\rho_z & (\rho_x^2 + \rho_y^2)/\rho_z^2 \end{bmatrix}. \tag{22}$$

Cox (personal communication) changed to this form for isopycnal mixing rather than the form given in Cox (1987).

b. A simple choice for F

With this choice for $\mathbf{F}$, Eq. (15), Q takes the form

$$Q = \nabla_z \cdot (\kappa \nabla_z \rho) - \frac{\partial}{\partial z}(\kappa \nabla_z \rho \cdot \nabla_z \rho / \rho_z). \tag{23}$$

It can be shown that the form (23) preserves all moments of ρ in domain average. Substituting this choice for $\mathbf{F}$ into Eq. (19) gives the following flux form for the passive scalar equation

$$\frac{D\tau}{Dt} + \nabla_z \cdot \left[\tau \frac{\partial}{\partial z}(\kappa \nabla_z \rho / \rho_z)\right] - \frac{\partial}{\partial z}[\tau \nabla_z \cdot (\kappa \nabla_z \rho / \rho_z)] = R(\tau). \tag{24}$$

Comparing Eqs. (2) and (19) or (24) again shows that the extra terms in the non-eddy-resolving model take the form of additional advections or fluxes of τ.

4. Discussion

We now discuss implementation of our mixing parameterization. If compressibility effects are ignored and density is assumed to be a linear function of temperature and salinity, then

$$\delta\rho/\rho_0 = \beta\delta S - \alpha\delta T, \tag{25}$$

where α and β are the coefficients of thermal expansion and saline contraction, assumed to be constants. Any other, linearly independent combination of δT and δS can be considered as a passive scalar, $\delta\tau$, on an isopycnal surface. These equations can be inverted to give δT and δS as linear combinations of $\delta\rho$ and $\delta\tau$. However, property C from section 2a, that ρ satisfies the scalar equation identically, was retained in the non-eddy-resolving model so that the active tracers T and S also satisfy the passive scalar equation in both the non-eddy-resolving and eddy-resolving models. Thus, the density equation (17) can be replaced by passive scalar equations for T and S, with the density gradients in Eq. (19) or (24) evaluated from Eq. (25).

In reality, the density of sea water is a very complicated function of pressure, p, T, and S. This function is used in the most comprehensive ocean circulation models. In this situation the eddy mixing should be along local potential density, σ, or local neutral surfaces rather than isopycnal surfaces. These surfaces are defined by

$$\delta\sigma/\sigma = \delta\rho/\rho - \gamma\delta p = \beta\delta S - \alpha\delta\theta, \tag{26}$$

where γ is the compressibility and θ is the potential temperature. Thus, in our formulation, isopycnal mixing is automatically changed to neutral mixing merely by using θ instead of T and letting α and β be functions of p, θ and S. Thus, with the general equation of state,

we propose the passive scalar equation (14), (19), or (24) for θ and S with the local gradients of the mixing surface evaluated by the right hand side of equation (26).

Global potential density surfaces depend upon the depth of the reference pressure, and McDougall and Jackett (1988) shows that neutral surfaces are not globally unique. In addition, McDougall (1987b) discusses two other effects that can occur when α and β are functions of p, θ and S. These are cabbeling and the very small thermobaric effect. Cabbeling occurs when two water masses at the same density mix their θ and S values which produces denser water. Thus, cabbeling and thermobaricity can produce diapycnal or dianeutral fluxes. For these reasons, we think it inappropriate to use the full equation of state for seawater in adiabatic or quasi-adiabatic models. It should only be used in diabatic models that have diapycnal or dianeutral fluxes in addition to isopycnal mixing.

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APPENDIX

Quasi-geostrophic Flow in a Zonally Uniform Channel: Momentum Flux and Energy Conversion in Isopycnal Coordinates

The primitive equations in isopycnal coordinates are

$$\frac{D\mathbf{u}}{Dt} + f\mathbf{k}\times\mathbf{u} + \nabla_\rho\phi = 0, \tag{A1}$$

$$\frac{\partial^2 h}{\partial t\partial\rho} + \nabla_\rho\cdot\left(\frac{\partial h}{\partial\rho}\mathbf{u}\right) = 0, \tag{A2}$$

$$h + \frac{\partial\phi}{\partial\rho} = 0, \tag{A3}$$

where $\mathbf{k}$ is the unit vertical vector and ϕ is the Montgomery potential. These equations are satisfied by a zonally and time independent mean flow of the form

$$h = \bar{h}(\rho, y), \quad \phi = \bar{\phi}(\rho, y),$$
$$u = \bar{u}(\rho, y), \quad v = 0, \tag{A4}$$

where the overbar represents an average over x and t. The full four-dimensional flow can be written as

$$h = \bar{h} + h',$$
$$\phi = \bar{\phi} + \phi',$$
$$u = \bar{u}_g + R\bar{u}_a + u',$$
$$v = R\bar{v}_a + v', \tag{A5}$$

where the x and t independent flow has been split into geostrophic and ageostrophic components denoted by subscripts g and a, respectively. By the usual midlatitude scaling argument, the ageostrophic component is only order R times as large as the geostrophic component where R is the Rossby number of the flow. The continuity equation (A2) averaged over x and t using (A5) gives

$$\frac{\partial}{\partial y}\left[\frac{\partial\bar{h}}{\partial\rho}\bar{v}_a + \overline{\frac{\partial h'}{\partial\rho}v'}\right] = 0. \tag{A6}$$

Integrating (A6) and using the boundary condition of no normal flow at the channel boundaries yields

$$\frac{\partial\bar{h}}{\partial\rho}\bar{v}_a = -\overline{\frac{\partial h'}{\partial\rho}v'} = \frac{\partial}{\partial\rho}\left(\kappa\frac{\partial\bar{h}}{\partial y}\right), \tag{A7}$$

where the second equality uses our parameterization given in Eq. (15). Using the geostrophic relations gives, to leading order in R,

$$\frac{\partial\bar{h}}{\partial\rho}\bar{v}_a = -\frac{\partial}{\partial\rho}\left(\kappa\frac{\partial^2\bar{\phi}}{\partial y\partial\rho}\right) = \frac{\partial}{\partial\rho}\left(\kappa f_0\frac{\partial\bar{u}_g}{\partial\rho}\right) + O(R^2), \tag{A8}$$

where f_0 is the average value of the Coriolis frequency f in the channel. Equations (A1) and (A2) can be combined to give

$$\frac{\partial}{\partial t}\left(\frac{\partial h}{\partial\rho}u\right) = -\nabla_\rho\cdot\left(\frac{\partial h}{\partial\rho}u\mathbf{u}\right) + \frac{\partial h}{\partial\rho}\left(fv - \frac{\partial\phi}{\partial x}\right). \tag{A9}$$

Taking the x and t average of (A9) yields

$$\frac{\partial}{\partial t}\left(\overline{\frac{\partial h}{\partial\rho}u}\right) = 0$$
$$= \overline{-\nabla_\rho\cdot\left(\frac{\partial h}{\partial\rho}u\mathbf{u}\right)} + f_0\frac{\partial\bar{h}}{\partial\rho}\bar{v}_a + O(R^2), \tag{A10}$$
$$= \overline{-\nabla_\rho\cdot\left(\frac{\partial h}{\partial\rho}u\mathbf{u}\right)} + \frac{\partial}{\partial\rho}\left(\kappa f_0^2\frac{\partial\bar{u}_g}{\partial\rho}\right) + O(R^2), \tag{A11}$$

using (A8). Thus, when κ is positive, our parameterization (15) acts as a downgradient vertical diffusion of mean (large-scale) momentum.

We now consider the rate of change of potential energy,

$$\frac{\partial}{\partial t}\iiint\tfrac{1}{2}h^2\,dxdyd\rho = \iiint\phi\frac{\partial^2 h}{\partial t\partial\rho}\,dxdyd\rho. \tag{A12}$$

Averaging over x and t and using the continuity equation (A2) in its averaged form (A6) gives

$$\frac{\partial}{\partial t}\int\int \frac{1}{2}\bar{h}^2 dyd\rho = 0$$

$$= \int\int \bar{\phi}\frac{\partial}{\partial y}\left[\frac{\partial \bar{h}}{\partial \rho}\bar{v}_a + \overline{\frac{\partial h'}{\partial \rho}v'}\right]dyd\rho, \quad \text{(A13)}$$

$$= -\int\int f_0\bar{u}_g\left[\frac{\partial \bar{h}}{\partial \rho}\bar{v}_a + \overline{\frac{\partial h'}{\partial \rho}v'}\right]dyd\rho + O(R^3), \quad \text{(A14)}$$

by an integration by parts and using the geostrophic relation. In an x and t average the energy conversion from mean kinetic energy to mean potential energy is equal to the conversion from mean potential energy to perturbation potential energy. This latter conversion is given to leading order by

$$-\int\int f_0\bar{u}_g\overline{\frac{\partial h'}{\partial \rho}v'}dyd\rho = \int\int f_0\bar{u}_g\frac{\partial}{\partial \rho}\left(\kappa\frac{\partial \bar{h}}{\partial y}\right)dyd\rho, \quad \text{(A15)}$$

using our parameterization (15). An integration by parts and use of the geostrophic relation gives the conversion from mean to perturbation potential energy as

$$-\int\int f_0\bar{u}_g\overline{\frac{\partial h'}{\partial \rho}v'}dyd\rho = -\int\int f_0^2\kappa\left(\frac{\partial \bar{u}_g}{\partial \rho}\right)^2 dyd\rho + O(R^3). \quad \text{(A16)}$$

Thus, when κ is positive, our parameterization (15) leads to a conversion of mean potential energy to eddy (subgrid-scale) potential energy.

REFERENCES

Cox, M. D., 1987: Isopycnal diffusion in a z-coordinate ocean model. *Ocean Modelling,* **74,** 1–5.

Holland, W. R., 1978: The role of mesoscale eddies in the general circulation of the ocean—numerical experiments using a wind-driven quasi-geostrophic model. *J. Phys. Oceanogr.,* **8,** 363–392.

Iselin, C. O., 1939: The influence of vertical and lateral turbulence on the characteristics of the waters at mid-depths. *Eos, Trans., AGU,* **20,** 414–417.

Kirwan, A. D., 1969: Formulation of constitutive equations for large-scale turbulent mixing. *J. Geophys. Res.,* **74,** 6953–6959.

McDougall, T. J., 1987a: Neutral surfaces. *J. Phys. Oceanogr.,* **17,** 1950–1964.

——, 1987b: Thermobaricity, cabbeling and water-mass conversion. *J. Geophys. Res.,* **92,** 5448–5464.

——, and D. R. Jackett, 1988: On the helical nature of neutral trajectories in the ocean. *Prog. in Oceanogr.,* **20,** 153–183.

McWilliams, J. C., and J. H. S. Chow, 1981: Equilibrium geostrophic turbulence. I. A reference solution in a β-plane channel. *J. Phys. Oceanogr.,* **11,** 921–949.

——, N. J. Norton, P. R. Gent and D. B. Haidvogel, 1989: A linear balance model of wind-driven, midlatitude ocean circulation. *J. Phys. Oceanogr.,* submitted.

Montgomery, R. B., 1940: The present evidence on the importance of lateral mixing processes in the ocean. *Bull. Amer. Meteor. Soc.,* **21,** 87–94.

Redi, M. H., 1982: Oceanic isopycnal mixing by coordinate rotation. *J. Phys. Oceanogr.,* **12,** 1154–1158.

Solomon, H., 1971: On the representation of isentropic mixing in ocean circulation models. *J. Phys. Oceanogr.,* **1,** 223–234.