**SIO 214A Homework 2**

Please hand in on paper, in class, on Tuesday, October 19.

1.) Show that the eqns for a perfect barotropic fluid, namely

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p \tag{2}$$

$$p = F(\rho) \tag{3}$$

conserve energy in the form

$$\frac{d}{dt} \iiint dxdydz \left( \frac{1}{2} \rho \mathbf{u} \cdot \mathbf{u} + \rho E(\rho) \right) = 0 \tag{4}$$

where the integral is over a rigid container on whose boundary the normal velocity vanishes ($\mathbf{u} \cdot \hat{\mathbf{n}} = 0$); and the internal energy per unit mass $E(\rho)$ is related to $F(\rho)$ by the equation

$$F(\rho) = \rho^2 \frac{d}{d\rho} E(\rho) \tag{5}$$

Relate (5) to an equation you may have seen in your thermodynamics book:

$$dE = -PdV + TdS \tag{6}$$

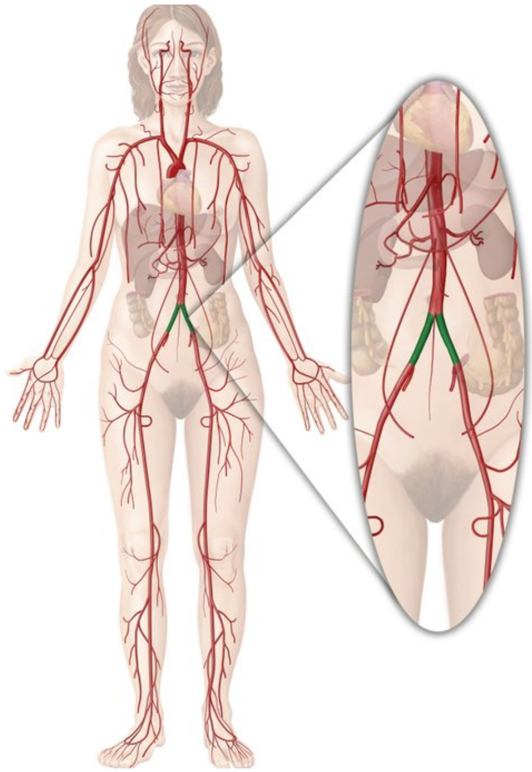
Find the corresponding energy equation for the linear equations

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{u} = 0 \tag{7}$$

$$\rho_0 \frac{\partial \mathbf{u}}{\partial t} = -\nabla p' \tag{8}$$

$$p' = c^2 \rho' \tag{9}$$

Relate the internal energy that appears in that equation to $E(\rho)$ in (4).

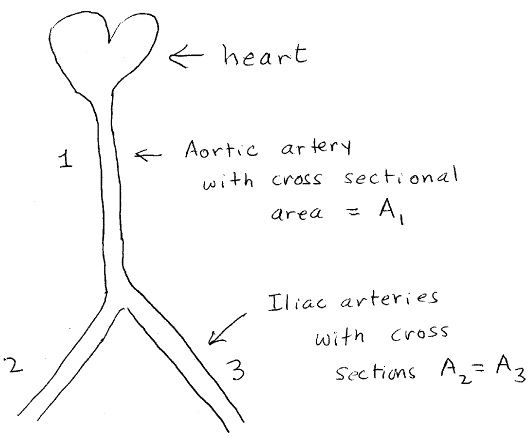


2.) Heart contractions send pressure pulses (sound waves!) down the aortic artery (see the figures) to the iliac bifurcation, where the pulses are transmitted to the two iliac arteries. The incident wave from the heart generally causes a reflected wave (back to the heart), besides the waves transmitted downwards through the iliac arteries to each leg. Assuming that the incident wave corresponds to a pressure pulse of the form

$$p_I(x,t) = F(t - x/c), \tag{10}$$

where the function $F(\cdot)$ is set by the heart, calculate the pressures $p_2(x,t)$ and $p_3(x,t)$ of the sound waves in the iliac arteries and the pressure $p_R(x,t)$ in the reflected wave. Assume that the cross-sectional area of the aortic artery is $A_1$, and that the iliac arteries have equal cross-sectional areas $A_2 = A_3$, but do not necessarily assume that $A_1 = A_2 + A_3$. (In the heart, $A_2 + A_3 < A_1$.) Here, $x$ is the distance traveled along each artery in the direction away from the heart, and $x = 0$ corresponds to the bifurcation.

Calculate the blood velocities in all three arteries.




The matching conditions at the junction are critical. Explain why they should be

$$p_I(0,t) + p_R(0,t) = p_2(0,t) = p_3(0,t) \tag{11}$$

and

$$A_1(u_1(0,t) + u_R(0,t)) = A_2 u_2(0,t) + A_3 u_3(0,t) \tag{12}$$

Show that the downward energy flux in the aorta is equal to the sum of the downward energy fluxes in the two iliac arteries.