

SIO203B/MAE294B Mid-term 2016

This exam is open notes, but no computers, iPhones or electronic assistance.

Problem 1

Find two terms in the $x \rightarrow \infty$ asymptotic expansion of

$$A(x) \stackrel{\text{def}}{=} \int_0^x t^{-1/3} e^{-t} dt. \quad (1)$$

Justify the asymptoticness of your two-term expansion by showing that remainder is negligible relative to the second term as $x \rightarrow \infty$.

Problem 2

Find the leading order uniformly valid $\epsilon \rightarrow 0$ approximation to the solution of

$$-(e^x g)_x = \epsilon g_{xx} + e^x, \quad \text{with BCs } g(0) = g(1) = 0. \quad (2)$$

Problem 3

Find two terms in the $\epsilon \rightarrow 0$ approximation to all roots of the polynomial $\epsilon z^4 - z + 1 = 0$.

Problem 4

$x(t)$ is defined via the initial value problem

$$\frac{dx}{dt} = x^2 - 4 + \frac{1}{23} \ln \frac{1}{x}, \quad \text{with IC } x(0) = 1. \quad (3)$$

Estimate $\lim_{t \rightarrow \infty} x(t)$ with one significant figure.

Problem 5

Find a leading order $x \rightarrow \infty$ approximation to

$$F(x) = \int_{-\infty}^{\infty} e^{+xt-t^4/4} dt. \quad (4)$$

(There is no need to justify asymptoticness.)