

SIO203B/MAE294B Mid-term 2017

This exam is open notes, but no computers, iPhones or electronic assistance.
All problems have equal weight.

Problem 1

The red army, with strength $R(t)$, fights the green army, with strength $G(t)$. The conflict starts from an initial condition $G(0) = 9$ and $R(0) = 5$ and proceeds according to

$$\dot{R} = -G, \quad \dot{G} = -2RG. \quad (1)$$

The war stops when one army is extinct. Which army wins, and what is the strength of the victorious army?

Problem 2

Find two-term approximations to all roots of

$$\epsilon x^3 + (x - 1)^2 = 0. \quad (2)$$

Problem 3

Find a leading-order approximate solution of the boundary value problem

$$\epsilon \frac{d^2 u}{dx^2} - u = -\cos\left(\frac{\pi\sqrt{x}}{2}\right), \quad (3)$$

posed on $0 < x < 1$ with boundary conditions $u(0) = u(1) = 0$.

Turn the page for problem 4

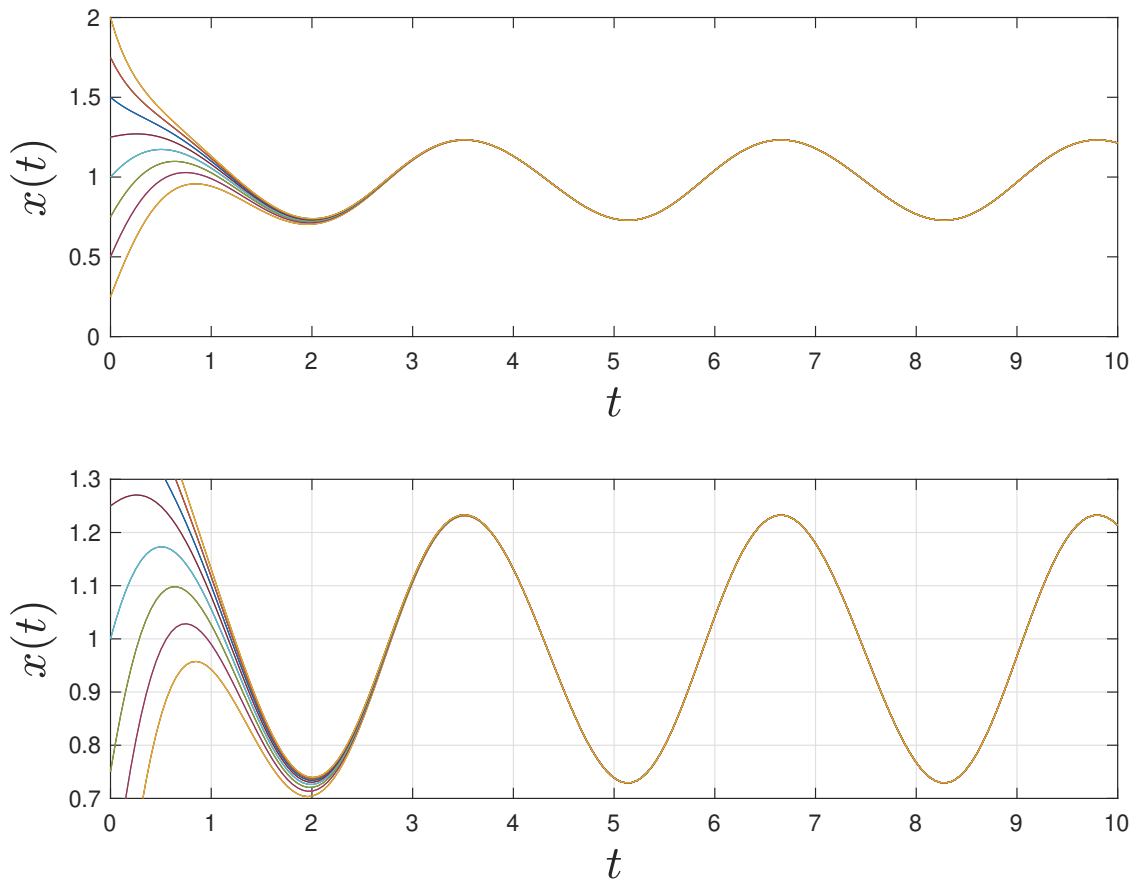


Figure 1: Evolution of 7 initial conditions. The bottom panel is an expanded view showing small oscillations about $x = 1$ at long time.

Problem 4

Figure 1 shows solutions of one of the following differential equations

$$\frac{dp}{dt} = 1 - p^2 + \epsilon \cos t, \quad \frac{dq}{dt} = 1 - q^2 + \epsilon \cos 2t, \quad (4)$$

$$\frac{dr}{dt} = -1 + r^2 + \epsilon \cos t, \quad \frac{ds}{dt} = -1 + s^2 + \epsilon \cos 2t. \quad (5)$$

The parameter ϵ is a smallish positive number. (i) Without explicitly solving a differential equation, determine which equation has been solved to produce figure 1? (ii) Estimate the value of ϵ used to make the figure.